Problem 1. a)

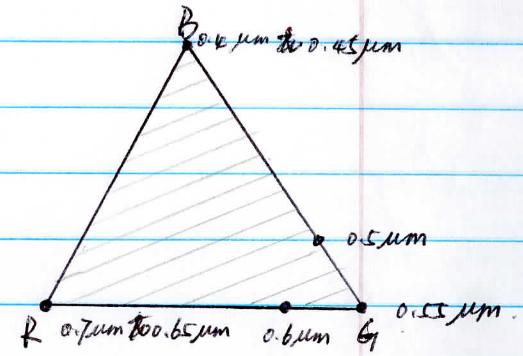
$$R_S = \int_{0.4}^{0.7} S(\lambda) Q_R(\lambda)\, d\lambda = 0.05 \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{120} \doteq 0.0083$$

$$G_S = \int_{0.4}^{0.7} S(\lambda) Q_G(\lambda)\, d\lambda = \frac{1}{10} = 0.1$$

$$B_S = \int_{0.4}^{0.7} S(\lambda) Q_B(\lambda)\, d\lambda = 0.15 \times 1 \times \frac{1}{2} = \frac{3}{40} \doteq 0.075 \quad \#$$

b)

λ (μm)	(r, g, b)
0.4	(0, 0, 1)
0.45	(0, 0, 1)
0.5	$(0, \frac{3}{4}, \frac{1}{4})$
0.55	(0, 1, 0)
0.6	$(\frac{1}{4}, \frac{3}{4}, 0)$
0.65	(1, 0, 0)
0.7	(1, 0, 0)



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c)

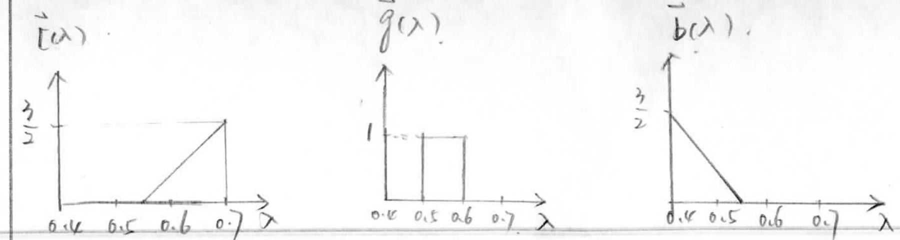
$P_R(\lambda) = \delta(\lambda - 0.65)$, $P_G(\lambda) = \delta(\lambda - 0.55)$, $P_B(\lambda) = \delta(\lambda - 0.45)$

$A = [a_{ij}]$, $a_{ij} = \int P_j(\lambda) Q_i(\lambda)\, d\lambda$.

$$A = \begin{bmatrix} Q_R(0.65) & Q_R(0.55) & Q_R(0.45) \\ Q_G(0.65) & Q_G(0.55) & Q_G(0.45) \\ Q_B(0.65) & Q_B(0.55) & Q_B(0.45) \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} \end{bmatrix}$$

$$p = A^{-1} C_T = \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{120} \\ \frac{1}{10} \\ \frac{3}{40} \end{bmatrix} = \begin{bmatrix} \frac{1}{80} \\ \frac{1}{10} \\ \frac{9}{80} \end{bmatrix}$$

$$\begin{bmatrix} P_R \\ P_G \\ P_B \end{bmatrix} = \begin{bmatrix} \frac{1}{80} \\ \frac{1}{10} \\ \frac{9}{80} \end{bmatrix} \quad \#$$



d/ $$\begin{bmatrix} \vec{r}(\lambda) \\ \vec{g}(\lambda) \\ \vec{b}(\lambda) \end{bmatrix} = A^{-1} \begin{bmatrix} Q_R(\lambda) \\ Q_G(\lambda) \\ Q_B(\lambda) \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} Q_R(\lambda) \\ Q_G(\lambda) \\ Q_B(\lambda) \end{bmatrix} = \begin{bmatrix} \frac{3}{2} Q_R(\lambda) \\ Q_G(\lambda) \\ \frac{3}{2} Q_B(\lambda) \end{bmatrix}$$

e/ $$\begin{bmatrix} p'_R \\ p'_G \\ p'_B \end{bmatrix} = \int s(\lambda) \begin{bmatrix} r(\lambda) \\ g(\lambda) \\ b(\lambda) \end{bmatrix} d\lambda = \begin{bmatrix} \frac{1}{80} \\ \frac{1}{10} \\ \frac{9}{80} \end{bmatrix}$$

Problem 2

(a)

$$\vec{r} = S^T \vec{n}$$

$$= \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3+1+0.5 \\ 1.5+1+1 \end{pmatrix} = \begin{pmatrix} 4.5 \\ 3.5 \end{pmatrix}$$

(b)

$$\vec{n}^* = S(S^TS)^{-1}S^T\vec{n}$$

$$S^TS^{-1} = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.5 \\ 0.5 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1.5 & 1.25 \\ 1.25 & 1.5 \end{pmatrix}$$

$$(S^TS)^{-1} = \frac{1}{2.75} \begin{pmatrix} 6 & -5 \\ -5 & 6 \end{pmatrix}$$

$$\vec{n}^* = \frac{1}{2.75} \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.5 \\ 0.5 & 1 \end{pmatrix} \begin{pmatrix} 6 & -5 \\ -5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2.75} \begin{pmatrix} 8.75 \\ 4 \\ 3.25 \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 35 \\ 16 \\ 13 \end{pmatrix}$$

(c) $$\vec{n}^c = \vec{n} - \vec{n}^* = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} - \frac{1}{2.75} \begin{pmatrix} 8.75 \\ 4 \\ 3.25 \end{pmatrix}$$
$$= \frac{1}{11} \begin{pmatrix} -2 \\ 6 \\ -2 \end{pmatrix}$$

(d)

a metamer n'

The response of this sensor S to the stimulus $\vec{n}$ and $\vec{n}'$ should be the same, but $\vec{n} \neq \vec{n}'$

$$n' = \vec{n}^* + \alpha \vec{n^c} \quad \text{where } \alpha \in \mathbb{R}$$
$$= \frac{1}{11}\begin{pmatrix} 35 \\ 16 \\ 13 \end{pmatrix} + \alpha \cdot \frac{1}{11} \begin{pmatrix} -2 \\ 6 \\ -2 \end{pmatrix}$$
$$= \frac{1}{11} \begin{pmatrix} 35 - 2\alpha \\ 16 + 6\alpha \\ 13 - 2\alpha \end{pmatrix}$$

(e) $$R = S(S^T S)^{-1} S^T$$
$$= \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.5 \\ 0.5 & 1 \end{pmatrix} \left[\begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.5 \\ 0.5 & 1 \end{pmatrix} \right]^{-1} \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 \end{pmatrix}$$
$$= \frac{1}{2.75} \begin{pmatrix} 2.5 & 0.75 & -0.25 \\ 0.75 & 0.5 & 0.75 \\ -0.25 & 0.75 & 2.5 \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 10 & 3 & -1 \\ 3 & 2 & 3 \\ -1 & 3 & 10 \end{pmatrix}$$

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Problem #3 sample solution

(a)
$$f(x,y) = \frac{1}{2}\left(1+\cos\left(2\pi(x-y)10\right)\right)\cdot rep_{4,4}\left[rect\left(\frac{x}{2},\frac{y}{2}\right)\right]$$
$$= \frac{1}{2}\left(1+\cos\left(2\pi(x-y)10\right)\right)\cdot \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} rect\left(\frac{x-4k}{2},\frac{y-4l}{2}\right)$$
$$= \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} \frac{1}{2}\left(1+\cos\left(2\pi(x-y)10\right)\right)\cdot rect\left(\frac{x-4k}{2},\frac{y-4l}{2}\right)$$

(b)
$$F(u,v) = CSFT\left\{\frac{1}{2}\left(1+\cos\left(2\pi(x-y)10\right)\right)\right\} ** \; CSFT\left\{rep_{4,4}\left[rect\left(\frac{x}{2},\frac{y}{2}\right)\right]\right\}$$
$$= \left[\frac{1}{2}\delta(u,v)+\frac{1}{4}\left(\delta(u-10,v+10)+\delta(u+10,v-10)\right)\right] ** \frac{1}{16}comb_{\frac{1}{4},\frac{1}{4}}\left[4sinc(2u,2v)\right]$$
$$= \frac{1}{4}comb_{\frac{1}{4},\frac{1}{4}}\left[sinc(2u,2v) ** \left\{\frac{1}{2}\delta(u,v)+\frac{1}{4}\left(\delta(u-10,v+10)+\delta(u+10,v-10)\right)\right\}\right]$$
$$= \frac{1}{4}comb_{\frac{1}{4},\frac{1}{4}}\left[\frac{1}{2}sinc(2u,2v)+\frac{1}{4}sinc\left(2(u-10),2(v+10)\right)+\frac{1}{4}sinc\left(2(u+10),2(v-10)\right)\right]$$
$$= \frac{1}{16}comb_{\frac{1}{4},\frac{1}{4}}\left[2sinc(2u,2v)+sinc\left(2(u-10),2(v+10)\right)+sinc\left(2(u+10),2(v-10)\right)\right]$$
$$= \frac{1}{8}sinc(2u,2v)+\frac{1}{16}\left\{sinc\left(2(u-10),2(v+10)\right)+sinc\left(2(u+10),2(v-10)\right)\right\}\cdot\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\delta\left(u-\frac{k}{4},v-\frac{l}{4}\right)$$
$$= \frac{1}{16}\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\left[2sinc(2u,2v)+sinc\left(2(u-10),2(v+10)\right)+sinc\left(2(u+10),2(v-10)\right)\right]\delta\left(u-\frac{k}{4},v-\frac{l}{4}\right)$$

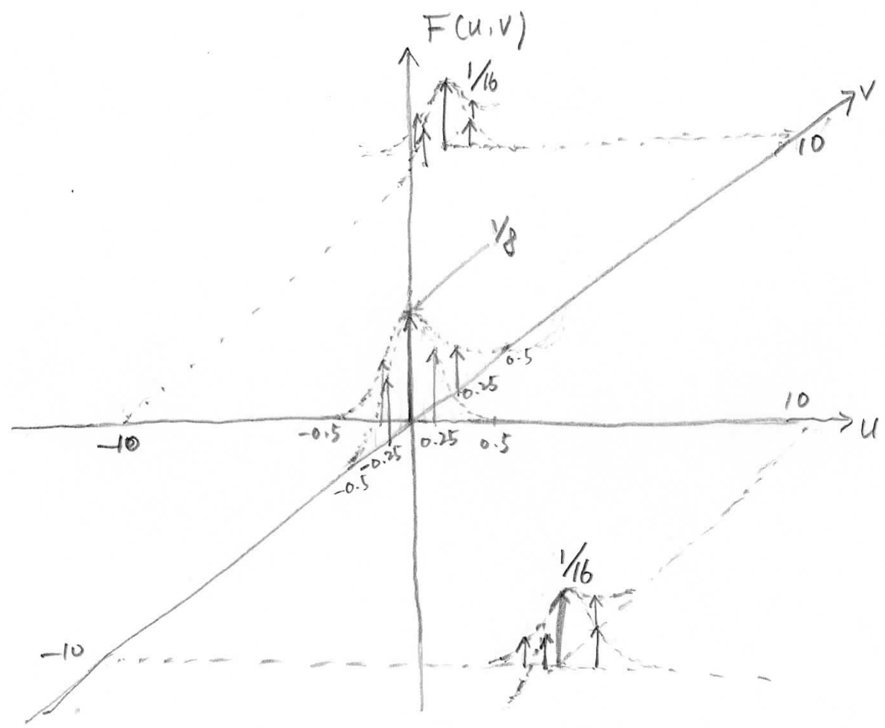
(d) $g(x,y) = f(x,y) \cdot \mathrm{rect}\left(\frac{x}{20}, \frac{y}{20}\right)$

$$G(u,v) = F(u,v) ** 400\,\mathrm{sinc}(20u, 20v)$$

$$= \frac{400}{16} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \left[2\,\mathrm{sinc}(2u,2v) + \mathrm{sinc}\left(2(u-10), 2(v+10)\right) + \mathrm{sinc}\left(2(u+10), 2(v-10)\right) \right] \cdot \mathrm{sinc}\left(20\left(u-\frac{k}{4}\right), 20\left(v-\frac{l}{4}\right)\right)$$

$$= 25 \sum_{k} \sum_{l} \left[2\,\mathrm{sinc}(2u,2v) + \mathrm{sinc}\left(2(u-10), 2(v+10)\right) + \mathrm{sinc}\left(2(u+10), 2(v-10)\right) \right] \cdot \mathrm{sinc}\left(20\left(u-\frac{k}{4}\right), 20\left(v-\frac{l}{4}\right)\right)$$

(c)



(e)