Corrected Solution for Problem 3(b) ECE 638 Exam No. 1 Fall 2013

We start with the second line of the posted solution

$$F(u,v)=\left[\frac{1}{2}\delta(u,v)+\frac{1}{4}\delta(u-10,v+10)+\frac{1}{4}\delta(u+10,v-10)\right] \\ **\frac{1}{16}\mathrm{comb}_{\frac{1}{4},\frac{1}{4}}\left[4\mathrm{sinc}(2\mathrm{u},2\mathrm{v})\right] \quad . \qquad (1)$$

Note that in the posted solution, there is a missing parenthesis to group the second and third impulse functions. What the author of the solution did next was to take the convolution operator inside the comb function. This seems questionable to me. At this point, I would have gotten rid of the comb operator before performing any convolutions to obtain

$$F(u,v)=\left[\frac{1}{2}\delta(u,v)+\frac{1}{4}\delta(u-10,v+10)+\frac{1}{4}\delta(u+10,v-10)\right] \\ **\frac{1}{4}\sum_k\sum_l \mathrm{sinc}(\frac{k}{2},\frac{l}{2})\delta(u-\frac{k}{4},v-\frac{l}{4}) \quad . \qquad (2)$$

Note that the sinc function is properly shown here as being sampled by the comb

$$\mathrm{sinc}(2\mathrm{u},2\mathrm{v})\big|_{u=\frac{k}{4},\,v=\frac{l}{4}}=\mathrm{sinc}\left(\frac{k}{2},\frac{l}{2}\right) \quad ; \qquad (3)$$

and it will stay that way throughout the rest of the derivation.

Now, we can apply our rule for the convolution of an impulse with something else, which is that we shift the something else to the location of the impulse. Here, we will apply that rule three times, once to each of the three impulses in the square brackets. The "something else" is everything to the right of the double asterisks. We thus obtain

$$F(u,v)=\frac{1}{8}\sum_k\sum_l \mathrm{sinc}(\frac{k}{2},\frac{l}{2})\delta(u-\frac{k}{4},v-\frac{l}{4}) \\ +\frac{1}{16}\sum_k\sum_l \mathrm{sinc}(\frac{k}{2},\frac{l}{2})\delta(u-\frac{k}{4}-10,v-\frac{l}{4}+10) \quad . \\ +\frac{1}{16}\sum_k\sum_l \mathrm{sinc}(\frac{k}{2},\frac{l}{2})\delta(u-\frac{k}{4}+10,v-\frac{l}{4}-10) \qquad (4)$$

This is the final answer for part (b); and it matches the picture shown in the posted solution for part (c). I did not bother to determine whether or not the expression in (4) matches that shown at the end of the posted solution for Problem 3(b), because what is shown in the posted solution is just the wrong way to approach the problem. It is difficult enough to think about doing things the right way, without wasting time to think about how to do them the wrong way.

The posted answer to part (d) seems more clearly to be incorrect, since the sinc function was never properly sampled. Here is the correct solution. We start with

$$g(x,y) = f(x,y)\mathrm{rect}\left(\frac{x}{20},\frac{y}{20}\right) \quad , \tag{5}$$

and

$$G(u,v) = F(u,v) ** 400\mathrm{sinc}(20u,20v) \quad , \tag{6}$$

as in the posted solution. But then we substitute the expression in (4) for $F(u,v)$ in (6), and again account for the convolution with impulses by replacing each impulse in (4) by a shifted $\mathrm{sinc}(20u,20v)$. Thus, we obtain for the final answer

$$\begin{aligned} G(u,v) = 50\sum_k\sum_l \mathrm{sinc}(\frac{k}{2},\frac{l}{2})\mathrm{sinc}\left(20\left(u-\frac{k}{4}\right),20\left(v-\frac{l}{4}\right)\right) \\ +25\sum_k\sum_l \mathrm{sinc}(\frac{k}{2},\frac{l}{2})\mathrm{sinc}\left(20\left(u-\frac{k}{4}-10\right),20\left(v-\frac{l}{4}+10\right)\right) \quad . \\ +25\sum_k\sum_l \mathrm{sinc}(\frac{k}{2},\frac{l}{2})\mathrm{sinc}\left(20\left(u-\frac{k}{4}+10\right),20\left(v-\frac{l}{4}-10\right)\right) \end{aligned} \tag{7}$$

The sketch posted in the solution for part (e) may not be entirely correct. At the level of detail shown, it is difficult to say for sure. But what we do want is to replace each impulse in the sketch for part (c) by an appropriately shifted $\mathrm{sinc}(20u,20v)$ with amplitude modulated by the samples $\mathrm{sinc}(\frac{k}{2},\frac{l}{2})$ of the sinc function and the constant scaling factor. Note that the entire main lobe of $\mathrm{sinc}(20u,20v)$ is only 1/10 units wide in spatial frequency in both the u and v directions, whereas the spacing between adjacent shifted replications of the sinc function is 1/4. So the shifted replications of $\mathrm{sinc}(20u,20v)$ should be well separated.