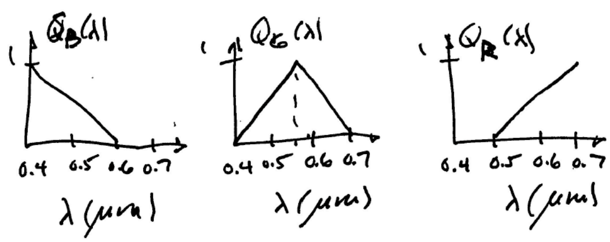
12 December 2011 **Name:___________________**

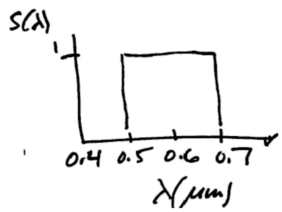
ECE 638 **Final Exam** **Fall 2011**

This exam is due at 11:59 PM EST on Wednesday 14 December 2011. It should be submitted electronically via e-mail to jpallebach@gmail.com. You may use any written or computational resources that you wish to use. However, you may not consult with anyone other than the course instructor regarding your solution approach. Please note that these problems are ordered according to the sequence in which the related material was covered in the lectures. They are not ordered according to my estimate of the difficulty of the problems. This exam contains 5 problems, and is worth 160 points: 1 – 35 pts, 2 – 40 pts., 3 – 30 pts, 4 – 35 pts, and 5 – 20 pts.

1. (35 pts.) Consider a three-channel sensor with the response functions $\left[Q_R(\lambda), Q_G(\lambda), Q_B(\lambda)\right]$ shown below.



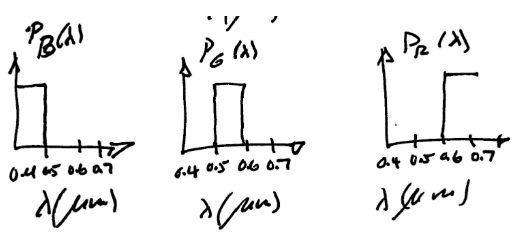
a. (8) Find the response of this sensor to the stimulus $S(\lambda)$ with the spectral power distribution shown below:



R = 0.1
G = 0.117
B = 0.025

Consider the primary set $\left[P_R(\lambda), P_G(\lambda), P_B(\lambda)\right]$ with power spectral distribution shown below:

$$A = \begin{bmatrix} 0.075 & 0.025 & 0 \\ 0.033 & 0.083 & 0.033 \\ 0 & 0.025 & 0.075 \end{bmatrix}$$



$$\begin{bmatrix} p_R \\ p_G \\ p_B \end{bmatrix} = A^{-1} \begin{bmatrix} R_S \\ G_S \\ B_S \end{bmatrix} = \begin{bmatrix} 0.9781 \\ 1.0656 \\ -0.0219 \end{bmatrix}$$

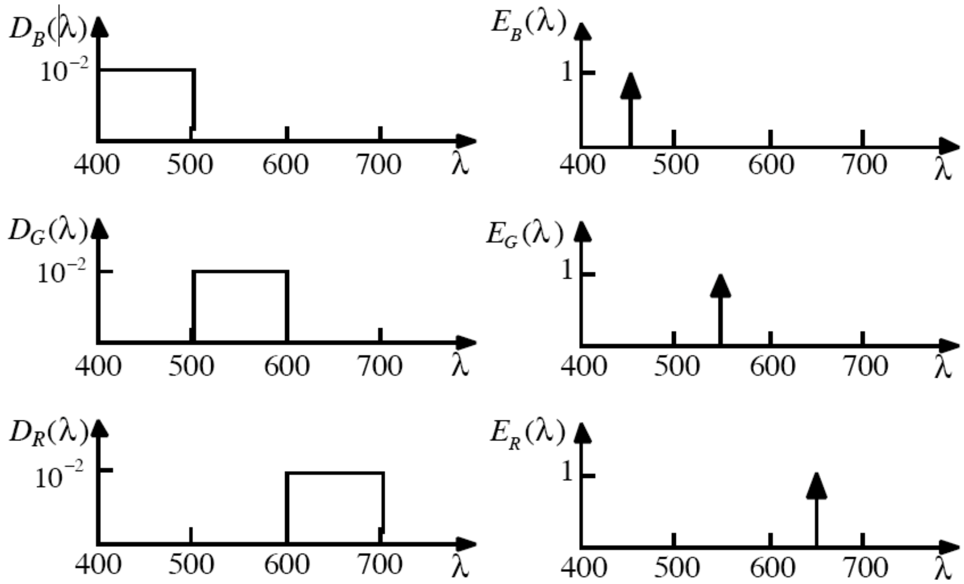
b. (10) Find the amounts of each of the three primaries that will yield a match to the stimulus $S(\lambda)$ from part a), as viewed by the sensor with response functions shown above.

c. (12) Find the color matching functions $\left[\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)\right]$ for this primary set.

d. (5) Use the color matching functions as an alternate solution to finding the amounts of the primaries that will match the stimulus $S(\lambda)$ shown above.

2. (40 pts.) You have two monitors ("D" and "E") with two different sets of primaries which have the spectral power distributions shown below. Assume that the two

monitors can generate stimuli consisting of combinations of their respective primaries using amounts R_D, G_D, B_D or R_E, G_E, B_E, respectively that are between 0 and 1.



a. (15) For given primary amounts R_D, G_D, B_D displayed on monitor D, find how much of primary amounts R_E, G_E, B_E should be displayed on monitor E; so that the two monitors yield a colorimetric match. (For this part, assume that both monitors have gamma 1.0 and no offset.

b. (6) Repeat part a. but find digital values between 0 and 255, assuming that the monitors have offsets and gammas as given below.

Monitor	Gamma	Offset
D	2.2	10
E	1.5	30

c. (4) Find the maximum luminances Y_D, Y_E displayable by the two monitors.

d. (10) Plot the gamuts of the two primaries on the CIE xy chromaticity diagram for a luminance equal to 50% of $\max(Y_D, Y_E)$. Be sure to show the spectral locus in your plots.

e. (5) Discuss how you would handle the gamut mismatch problem, i.e. describe explicitly what procedure you would use to display a color, for example on monitor D, which is in gamut on monitor E, but not in gamut for monitor D.

3. (30 pts.) Consider the final stage of sequential scalar quantization in which we must design a series of 1-D quantizers. Suppose that the conditional probability density $f_X(x)$ for one such quantizer is given by

$$f_X(x) = \begin{cases} 3x^2, & 0 \le x \le 1 \\ 0, & \text{else} \end{cases}$$

Assume that we are going to quantize the random variable X into just two levels, by splitting the density at level $X = \xi$.

a. (4) Find an expression for the exact mean-squared quantization error $E(\xi)$ as a function of the split location ξ.

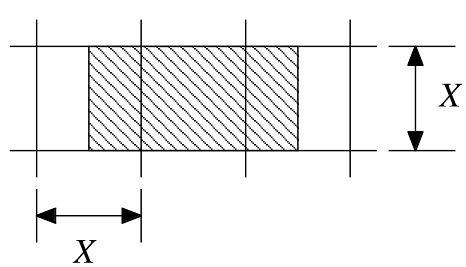
b. (10) Find the optimal value for ξ which minimizes $E(\xi)$, and compute the resulting mean-squared error.

c. (6) Assume that we choose ξ to simply split $f_X(x)$ into two units of equal area. Find ξ and use your answer to part a to find the resulting error.

d. (10) Assume that we choose ξ to split the asymptotically optimal grid point density function $\lambda(x) = [f_X(x)]^{1/3} / \int [f_X(x)]^{1/3} dx$ into two units of equal area. Find ξ and use your answer to part a to find the resulting error.

4. (35 pts.) This problem deals with halftoning. For this problem, assume that all continuous-tone and halftone images are represented in units of absorptance between 0 and 1.

In class, we discussed the hard circular dot overlap model as a method for representing dot gain in the printing process. Consider a printer with an addressable raster of points separated by X in the horizontal and vertical directions. Suppose that each printer dot has size $X \times 2X$ as shown in the figure below. Within the shaded area, the absorptance is 1. Where dots overlap, the absorptance is still 1. Where there is no dot coverage, the absorptance is 0.



With a screening process, the only way to compensate for dot gain is to preprocess the continuous-tone image with a tone-correction curve, or equivalently to adjust the thresholds according to the desired tone correction curve. The tone correction curve is a simple memoryless nonlinear operation applied to the gray values of the continuous-tone image before the halftoning process takes place.

Consider a screening process with the 4×2 threshold matrix given below.

1 / 16	5 / 16
13 / 16	9 / 16
7 / 16	3 / 16
11 / 16	15 / 16

a. (10) Find the appropriate tone correction curve for this halftoning algorithm.

Tone correction can be used with any halftoning algorithm. It always assures correct average tone; but it may result in visually poor textures. To overcome this problem, it is necessary to embed the dot gain model directly into the halftoning algorithm. One algorithm for which this can be done is error diffusion.

Consider the error diffusion algorithm defined by the equations below.

$$g[m,n] = \begin{cases} 1, & \tilde{f}[m,n] \geq 0.5 \\ 0, & \text{else} \end{cases}$$

$$e[m,n] = g[m,n] - \tilde{f}[m,n]$$

$$\tilde{f}[m,n+1] = \tilde{f}[m,n+1] - e[m,n]/2$$

$$\tilde{f}[m+1,n] = \tilde{f}[m+1,n] - e[m,n]/2$$

Here it is assumed that the image is processed from left to right (increasing n), and then top to bottom (increasing m). Relative to the pixel currently being binarized, half the error is diffused to the pixel ahead on the same line; and half the error is diffused to the pixel on the line below. Initially, the modified continuous-tone image $\tilde{f}[m,n]$ is identical to the original continuous-tone image $f[m,n]$ at all pixels. As we process the image, $\tilde{f}[m,n]$ is updated by the diffused error terms.

b. (10) Assume an ideal printer with $X \times X$ hard dots (i.e. no dot gain). Find the bit map that the error diffusion algorithm above would generate for a 2×4 continuous-tone image with constant value 0.5.

c. (15) Now assume the printer has an $X \times 2X$ hard dot as discussed at the beginning of this problem. Incorporate this model within the error diffusion algorithm by updating $\tilde{f}[m,n]$ immediately after $g[m,n]$ is determined, to account for the absorptance added if a dot was printed at $[m,n]$. This calculation is performed before computing the error to be diffused. Again find the bit map that the error diffusion algorithm would generate for a 2×4 continuous-tone image with constant value 0.5.

5. (20 pts.) This problem deals with color appearance models.

   a. (5) What are the specific limitations of colorimetry that color appearance models seek to address?

   b. (5) Describe a sample practical application of color appearance modeling.

   c. (5) Provide the overall block diagram and details of each component for the color appearance model that was discussed in class, and which uses the von-Kries model as the basis for the color appearance transformation.

   d. (5) In addition, to the detailed functional description of each stage in the color appearance model, discuss, from a high-level and intuitive viewpoint, what each stage is actually doing.