

very nice!

135/135

ECE 638 Final Exam, Dec. 2009  
Satyam Srivastava

Prob 1:

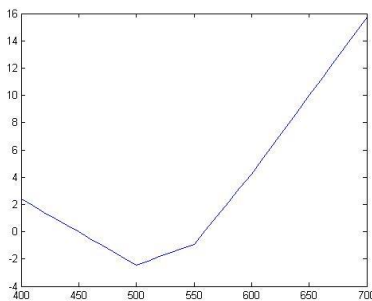
(a)  $R = (1/2) * 0.2 * 1 = 0.100$   
 $G = 0.1167$   
 $B = 0.025$

(b)  $A =$

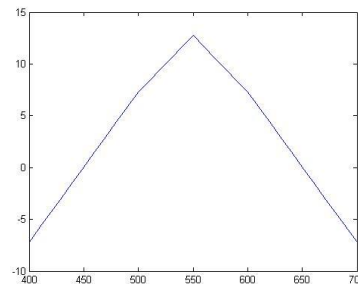
0.075	0.025	0.000
0.033	0.083	0.033
0.000	0.025	0.075

$$p = A^{-1} [R \ G \ B]^T$$
$$= [0.9962 \ 1.0115 \ -0.0038]^T$$

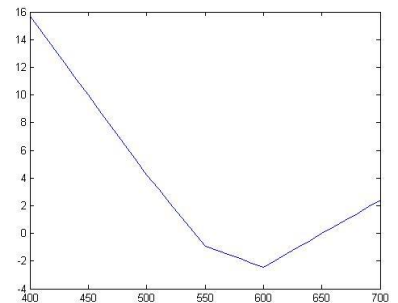
(c)  $[r(\mu) \ g(\mu) \ b(\mu)] = A^{-1} [Q_R(\mu) \ Q_G(\mu) \ Q_B(\mu)]^T$   
These are plotted next:



$r(\mu)$



$g(\mu)$



$b(\mu)$

(d)  $p = [\int S(\mu) r(\mu) d\mu \quad \int S(\mu) g(\mu) d\mu \quad \int S(\mu) b(\mu) d\mu]^T$   
 $[106.2842 \ 101.1475 \ 2.9508]^T$

Note: The values in (b) and (d) differ in scale but also differ slightly in the relative sense. This is because of the handling of boundary values in continuous integration in (b) versus discrete summation in (d).

**Prob 3:**

(a) Using MATLAB, we determine the 5x5 output with the following code:

```
clear;

thres=0.5;
img=ones(5,5);
img = img .* 0.25;

for i=1:5
    for j=1:5
        oldval = img(i,j);
        if (img(i,j)<thres)
            img(i,j) = 0.0;
        else
            img(i,j) = 1.0;
        end
        err = oldval - img(i,j);
        if (i<5)
            img(i+1,j)=img(i+1,j) + 0.5*err;
        end
        if (j<5)
            img(i,j+1)=img(i,j+1) + 0.5*err;
        end
    end
end
```

The halftone image is:

img =

```
0 0 0 0 0
0 1 0 1 0
0 0 1 0 0
0 1 0 0 0
0 0 0 0 1
```

(b) Consider two pixels A and B where B is the bottom-right neighbor of A. Assuming that the pixel information is received in raster order, the error corresponding to the halftone operation at A can affect B as the farthest pixel. In other words, at the time of processing B, the processor must have the information of the errors obtained at (every preceding pixel up-to) A. Therefore, the processor would need a minimum of **W+1** words of memory.

**Prob 5:**

(a) Colorimetry assumes that two stimuli matching in the tristimulus values will appear identical to an observer under the given viewing conditions. However, the visual similarity may not hold if the viewing conditions change. Color appearance models attempt to predict how well the visual similarity under one viewing condition would be carried forward to another viewing condition.

(b) Color appearance models would be useful in the problem of matching the hardcopy and the softcopy of an image. It may also be useful in material science where the color of the product is important and the product may be used under different illuminations.

(d) At a higher level, the function of the different blocks can be described as follows:

*HPE*: This block estimates the response of the human eye (cones) when the given stimulus (specified by its XYZ value) is viewed under the given conditions.

*ADPT*: This block primarily tries to minimize the effect of the viewing conditions by normalizing the LMS values by the maximum values achieved under the given conditions. This is equivalent to normalizing by the LMS response corresponding to the scene white point.

*IHPE*: The adapted tristimulus values are obtained from the normalized LMS values using the inverse of the HPE matrix.

*IADPT*: Logically, the inverse of ADPT, this stage de-normalizes the LMS values under the new viewing conditions.

(c) Attached

Prob 2:

$$2(a) \quad f_Y(y) = \sum_{k=-\infty}^{\infty} p_k \delta(y - k\Delta)$$

$$\text{where } p_k = P_Y\{Y = k\Delta\} \\ = \int_{(k-\frac{1}{2})\Delta}^{(k+\frac{1}{2})\Delta} f_X(x) dx$$

Let  $p_k$  be represented as the value of another function  $h(x)$ , evaluated at  $k\Delta$  that is,

$$p_k = h(k\Delta)$$

This suggests that  $h(x)$  is of the form

$$h(x) = \int_{x_0 - \frac{\Delta}{2}}^{x_0 + \frac{\Delta}{2}} f_X(x) dx$$

$$\text{Let } h(x) = f_X(x) * w(x)$$

$$\text{where } w(x) = \text{rect}_{\Delta}(x)$$

$$= \begin{cases} 1, & |x| \leq \Delta/2 \\ 0, & |x| > \Delta/2 \end{cases}$$

Then  $h(k\Delta) = f_X(x) * w(x)$  evaluated at  $x = k\Delta$

$$\text{which gives } \int_{(k-\frac{1}{2})\Delta}^{(k+\frac{1}{2})\Delta} f_X(x) dx$$

$$\therefore f_Y(y) = \sum_{k=-\infty}^{\infty} h(k\Delta) \delta(y - k\Delta)$$

$$= \text{comb}_{\Delta} h(y)$$

$$= \text{comb}_{\Delta} [w(y) * f_X(y)]$$

2(b) We note that this definition  $\Phi_x(u)$  is the same as the Fourier transform of ~~X~~ the density of X.

$$\begin{aligned}\Phi_x(u) &= E\{e^{-j2\pi x}\} \\ &= \int_{-\infty}^{\infty} e^{-j2\pi x} f_x(x) dx\end{aligned}$$

From (a),  $f_y(y) = \text{comb}_{\Delta}[w(x) * f_x(x)]$

Then using the standard Fourier transform pair:

$$\text{comb}_T[x(t)] \Leftrightarrow \frac{1}{T} \text{rep}_{\frac{1}{T}}[X(f)]$$

with  $T = \Delta$ ,  $x(t) = h(t)$

$$\Phi_y(u) = \frac{1}{\Delta} \text{rep}_{\frac{1}{\Delta}}[H(u)]$$

Also using  $x(t) * y(t) \Leftrightarrow X(f) \cdot Y(f)$

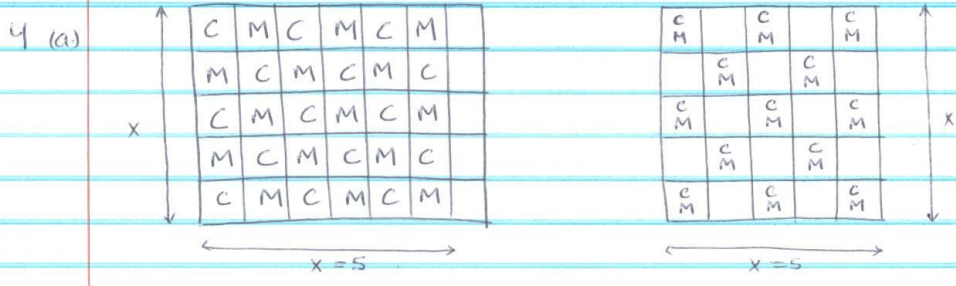
$$H(u) = W(u) \Phi_x(u)$$

$$\therefore \Phi_y(u) = \frac{1}{\Delta} \text{rep}_{\frac{1}{\Delta}}[W(u) \Phi_x(u)]$$

$$= \frac{1}{\Delta} \sum_{l=-\infty}^{\infty} W(u - \frac{l}{\Delta}) \Phi_x(u - \frac{l}{\Delta})$$

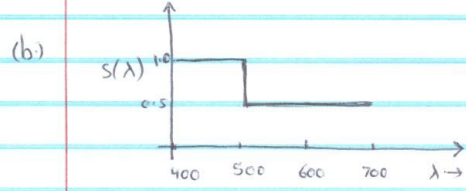
Note:  $W(u)$  is the characteristic function as per the problem definition, but  $w(x)$  is not random, thus we use  $W(u)$  as the Fourier transform of  $w(x)$

Prob 4:



Dot-off-dot

Dot-on-dot

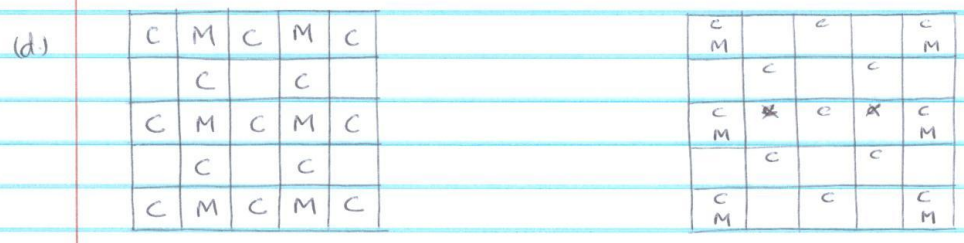


$S(\lambda)$  for both dot-off-dot and dot-on-dot

$$S_{\text{off}}(\lambda) = 0.5 P_c(\lambda) + 0.5 P_m(\lambda)$$

$$S_{\text{on}}(\lambda) = 0.5 [P_c(\lambda) \cdot P_m(\lambda)] + 0.5 P_w(\lambda)$$

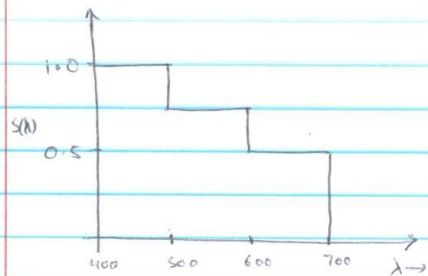
(c) The patch would be a light blue with a hint of red. A color produced on the computer with RGB values of (128, 128, 255) would be an approximation. This color appears similar to the Medium Slate Blue.





$$4(e) \quad S_{\text{off}}(\lambda) = 0.5 P_c(\lambda) + 0.25 P_m(\lambda) + 0.25 P_w(\lambda)$$

$$S_{\text{on}}(\lambda) = 0.25 P_c(\lambda) + 0.25 [P_c \cdot P_m(\lambda)] + 0.5 P_w(\lambda)$$

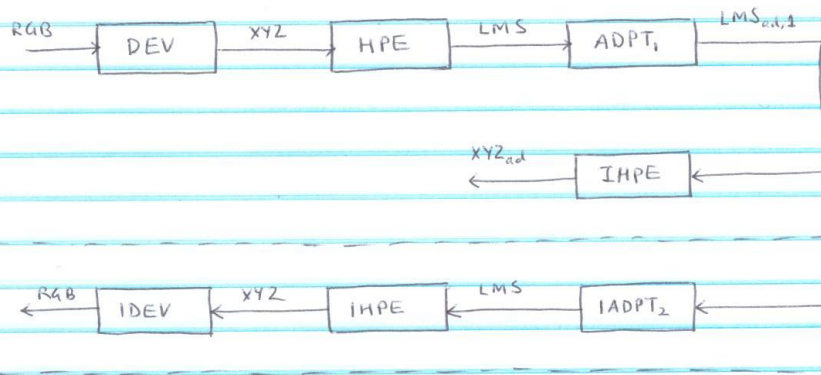


$S(\lambda)$  for both dot-off-dot  
and dot-on-dot

Note: Even with different area coverage for each  
tower dots, the shape of the spectrum for both  
methods remains the same.

Prob 5 (c):

5 (c)



DEV/IDEV : Forward and inverse device models. These are not part of the CAM but included for completeness.

HPE : Hunt-Pointer-Estevéz ~~trans~~ linear transformation

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \quad \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = M^{-1} \begin{bmatrix} L \\ M \\ S \end{bmatrix} \quad \text{in IHPE}$$

$$ADPT_1: \begin{bmatrix} L \\ M \\ S \end{bmatrix}_{ad,1} = \begin{bmatrix} 1/L_w & 0 & 0 \\ 0 & 1/M_w & 0 \\ 0 & 0 & 1/S_w \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

where  $[L, M, S]_w^T$  is the maximum LMS cone response under viewing condition 1.

Note: The blocks inside the dashed line are inverse of the CAM blocks and correspond to the transformation of the adapted LMS values to a different viewing condition.