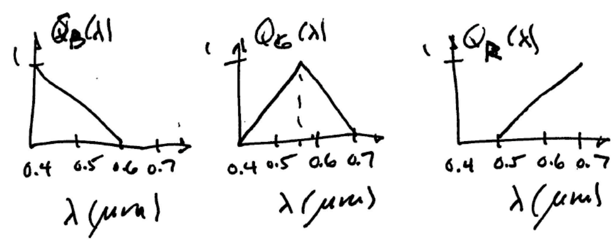
11 December 2009 **Name:___________________**

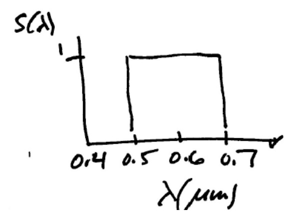
ECE 638 **Final Exam** **Fall 2009**

This exam is due no later than 10p EST on Friday 18 December 2009. It should be submitted electronically via e-mail to allebach@purdue.edu. You may use any written or computational resources that you wish to use. However, you may not consult with anyone other than the course instructor regarding your solution approach. Please note that these problems are ordered according to the sequence in which the related material was covered in the lectures. They are not ordered according to my estimate of the difficulty of the problems. This exam contains 5 problems, and is worth 135 points: 1 – 35 pts, 2 – 20 pts., 3 – 25 pts, 4 – 35 pts, and 5 – 20 pts.

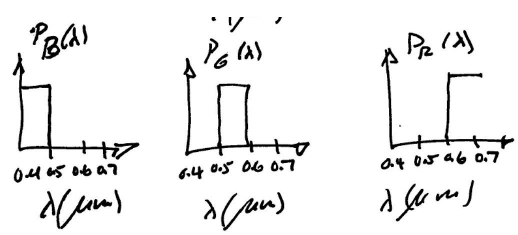
1. (35 pts.) Consider a three-channel sensor with the response functions $\left[Q_R(\lambda),Q_G(\lambda),Q_B(\lambda)\right]$ shown below.



   a. (8) Find the response of this sensor to the stimulus $S(\lambda)$ with the spectral power distribution shown below:



   Consider the primary set $\left[P_R(\lambda),P_G(\lambda),P_B(\lambda)\right]$ with power spectral distribution shown below:



   b. (10) Find the amounts of each of the three primaries that will yield a match to the stimulus $S(\lambda)$ from part a), as viewed by the sensor with response functions shown above.

   c. (12) Find the color matching functions $\left[\bar{r}(\lambda),\bar{g}(\lambda),\bar{b}(\lambda)\right]$ for this primary set.

   d. (5) Use the color matching functions as an alternate solution to finding the amounts of the primaries that will match the stimulus $S(\lambda)$ shown above.

2. (20 pts.) In class, we discussed quantization, which is important in many applications of imaging systems. Later, we also discussed sampling theory. The

purpose of this problem is to use sampling theory to analyze the effect of quantization.

For this problem, we will consider a uniform quantizer with step-size Δ. Let X denote the input to the quantizer; and let Y denote the output. The quantization operation can be expressed as $Y = Q(X) = k\Delta,$ where $(k-\frac{1}{2})\Delta < X \le (k+\frac{1}{2})\Delta$, and k is integer-valued.

In class, we discussed the fact that the quantizer error $D = Y - X$ can be approximated as a random variable that is uniformly distributed on the interval $[-\frac{1}{2}\Delta, \frac{1}{2}\Delta)$. Assume that X is a random variable with probability density function $f_X(x)$. The purpose of the first part of this problem is to find an exact expression for the probability density $f_Y(y)$ of the quantizer output in terms of $f_X(x)$. From the expression given in the preceding paragraph, it follows that

$$f_Y(y) = \sum_{k=-\infty}^{\infty} p_k \delta(y - k\Delta),$$

where

$$\begin{aligned} p_k &= \Pr\{Y = k\Delta\} \\ &= \int_{(k-\frac{1}{2})\Delta}^{(k+\frac{1}{2})\Delta} f_X(x)dx \end{aligned}.$$

Now, let see how we can use sampling theory to obtain an alternate expression for $f_Y(y)$ in terms of $f_X(x)$. Recall that we define the comb operator as

$$\text{comb}_T\left[f(x)\right] = \sum_{k=-\infty}^{\infty} f(kT)\delta(x - kT).$$

a. (10) Show that the density function for the quantizer output Y may be expressed as

$$f_Y(y) = \text{comb}_\Delta\left[w(x) * f_X(x)\right],$$

where $*$ denotes convolution and $w(x)$ is an appropriately chosen function. Determine what is the function $w(x)$.

Let

$$\varphi_X(u) = E\left\{e^{-j2\pi X}\right\}$$

denote the characteristic function for the random variable X, where $E\{\bullet\}$ denotes statistical expectation and u is the frequency variable.

b. (10) Show that the characteristic function for the output Y of the quantizer may be expressed in terms of the characteristic function of the input X, as

$$\varphi_Y(u) = \frac{1}{\Delta}\sum_{\ell=-\infty}^{\infty} W(u - \tfrac{\ell}{\Delta})\varphi_X(u - \tfrac{\ell}{\Delta})$$

3. (25 pts.) This problem deals with halftoning.

a. (20) Consider a 5×5 pixel constant, continuous-tone image patch with absorptance 0.25. Suppose we halftone this image, using 2-weight error diffusion, where $\frac{1}{2}$ the error is diffused to the next pixel ahead of the current pixel on the current scan line; and $\frac{1}{2}$ the error is diffused to the pixel immediately below the current pixel on the next scan line to be processed. Determine the 5×5 array of halftone image values.

b. (5) Suppose that we wish to halftone an image with size $H \times W$ pixels using an embedded processor to implement a standard 4-weight error diffusion. We receive the continuous-tone image values pixel-by-pixel over a bus; and as we generate the halftone values, we send them back out over the bus pixel-by-pixel. Determine the minimum amount of memory that the embedded processor needs to have to be able to perform the error diffusion.

4. (35 pts.) Consider a two-colorant system with colorants C and M. Suppose that these colorants have ideal block spectral reflectances given by

$$P_C(\lambda) = \begin{cases} 1, & 400 < \lambda \le 600, \\ 0, & \text{else.} \end{cases} \quad P_M(\lambda) = \begin{cases} 1, & 400 < \lambda \le 500, 600 < \lambda \le 700 \\ 0, & \text{else.} \end{cases}$$

These colorants are printed on ideal white media with spectral reflectance given by

$$P_W(\lambda) = \begin{cases} 1, & 400 < \lambda \le 700, \\ 0, & \text{else.} \end{cases}$$

Throughout this problem, the units of λ are nanometers. Suppose that the fractional area coverage of each of the two colorants C and M within a patch of size $X \times X$ is a_C and a_M, respectively.

Suppose we have an instrument that measures the spatially averaged spectral reflectance within this $X \times X$ patch. So the instrument returns the function

$$S(\lambda) = \frac{1}{X^2} \int_{-X/2}^{X/2} \int_{-X/2}^{X/2} R(x, y; \lambda)\, dx\, dy$$

where $R(x, y; \lambda)$ denotes the spectral reflectance of the printed patch as a function of the spatial coordinates (x, y), and thus is dependent on the halftoning algorithm that is used to print the patch.

We wish to consider two distinct special cases: (1) full *dot-off-dot* printing where there is no overlap of the two colorants anywhere within the $X \times X$, and (2) full *dot-on-dot* printing, where the two colorants overlap to the maximum extent possible, i.e. if $a_C < a_M$, then C is always printed with M, and never occurs alone; and if $a_M < a_C$, then M is always printed with C, and never occurs alone. For a given mixture (a_C, a_M), let $S_{\text{dot-off-dot}}(\lambda)$ and $S_{\text{dot-on-dot}}(\lambda)$ denote the spatially averaged spectral reflectances returned by our instrument for these two cases.

a. (5) Sketch typical dot-off-dot and dot-on-dot halftone patterns for the case where $a_C = a_M = 0.5$.

b. (10) Determine $S_{\text{dot-off-dot}}(\lambda)$ and $S_{\text{dot-on-dot}}(\lambda)$ for the case where $a_C = a_M = 0.5$.

c. (4) Assuming that the viewer cannot resolve the structure of the printed halftone patterns, i.e. the patches appear to have a single uniform color, how would you describe the visual appearance of the printed patch for the case.

d. (6) Sketch typical dot-off-dot and dot-on-dot halftone patterns for the case where $a_C = 0.5, a_M = 0.25$.

e. (10) Determine $S_{\text{dot-off-dot}}(\lambda)$ and $S_{\text{dot-on-dot}}(\lambda)$ for the case where $a_C = 0.5, a_M = 0.25$.

5. (20 pts.) This problem deals with color appearance models.

a. (5) What are the specific limitations of colorimetry that color appearance models seek to address?

b. (5) Describe a sample practical application of color appearance modeling.

c. (5) Provide the overall block diagram and details of each component for the color appearance model that was discussed in class, and which uses the von-Kries model as the basis for the color appearance transformation.

d. (5) In addition, to the detailed functional description of each stage in the color appearance model, discuss, from a high-level and intuitive viewpoint, what each stage is actually doing.