Analysis of Fourier Components in Virtual Haptic Gratings

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Square waves of a given magnitude can be defined as the sum of a series of sinusoidal waves of increasing frequency and decreasing magnitude.

We will analyze the detection thresholds of virtual sinusoidal and square wave gratings.

- Do tactile detection thresholds depend upon the fundamental frequencies of the waveforms?

We will analyze the sinusoidal and square wave discrimination thresholds to determine whether higher-order harmonics contribute to detectability.
Overview

- Background
- Waveform Definition/Fourier Series
- Apparatus
- Procedure
- Hypotheses
Background

- Morley, Goodwin, and Darian-Smith (1983)
  - Similar to our experiment 2 (discrimination)
    - 3 interval forced-decision experiment
  - Quantitative description of tactile discrimination using gratings
    - Alternating grooves and ridges
  - Using fingertips to rub the grating, the subject discriminates the spatial period of grating
  - 75% of time subjects can discriminate two gratings whose spatial period differ by 5.2%
Background

- Campbell & Robson (1968)
  - Measured contrast thresholds of gratings of various waveforms (sine, square, rectangular, saw-tooth).
  - To what extent can the contrast-sensitivity function be used to predict contrast thresholds?
  - Contrast sensitivity for square-wave grating is $4/\pi$ times $>$ contrast sensitivity for a sine-wave grating.
  - Found that sine-wave grating is perceived to be different from square-wave grating when the square-wave’s third harmonic reaches its threshold.
**Sinusoidal Wave**

- Defined as:
  \[ h_{\sin}(x) = A \cdot \sin(2\pi x/\lambda) \]
• Defined as:
  
  \[ h_{\text{square}}(x) = \begin{cases} 
  A & \text{if } \sin(2\pi x/\lambda) > 0 \\
  -A & \text{if } \sin(2\pi x/\lambda) < 0 
  \end{cases} \]

• Fourier series:
  
  \[ h_{\text{square}}(x) = A \cdot \frac{4}{\pi} \left[ \sin(2\pi x/\lambda) + \frac{1}{3}\sin(3 \cdot 2\pi x/\lambda) + \frac{1}{5}\sin(5 \cdot 2\pi x/\lambda) + \ldots \right] \]
Waveform Definition

- **Fundamental Frequency**: $A \times \frac{4}{\pi} \sin\left(\frac{2\pi x}{\lambda}\right)$
- **3rd Harmonic**: $A \times \frac{4}{3\pi} \sin\left(3 \times \frac{2\pi x}{\lambda}\right)$
- **5th Harmonic**: $A \times \frac{4}{5\pi} \sin\left(5 \times \frac{2\pi x}{\lambda}\right)$
Waveform Definition
Methods - Apparatus

- Mini-stick
  - 3-DOF
  - 1µm position resolution
  - Force commands updated at 2kHz
Methods - Procedure

- Three-interval 1-up 3-down adaptive tracking procedure

![Graph showing transformed up-down staircase with positive responses indicated by black squares and negative responses indicated by white circles. The graph depicts the stimulus level in decibels (db) against trial number, with a downward trend indicating adaptation. The reference is Leek (2001).]
Hypotheses

- Lower detection thresholds for square-wave gratings than for sinusoidal gratings
  - Detection threshold for square waves will be $4/\pi$ lower than the detection threshold for the corresponding sinusoidal waves

- Discrimination thresholds for sinusoidal and square wave gratings will be similar to the detection thresholds for sinusoidal gratings translated by a factor of three in spatial frequency and threshold amplitude
  - Due to the contribution of the third harmonic