

# Multidimensional Scaling

# Outline

- **Motivation**
- **An Example on Haptic Texture Perception**
- **Summary**

# Motivation

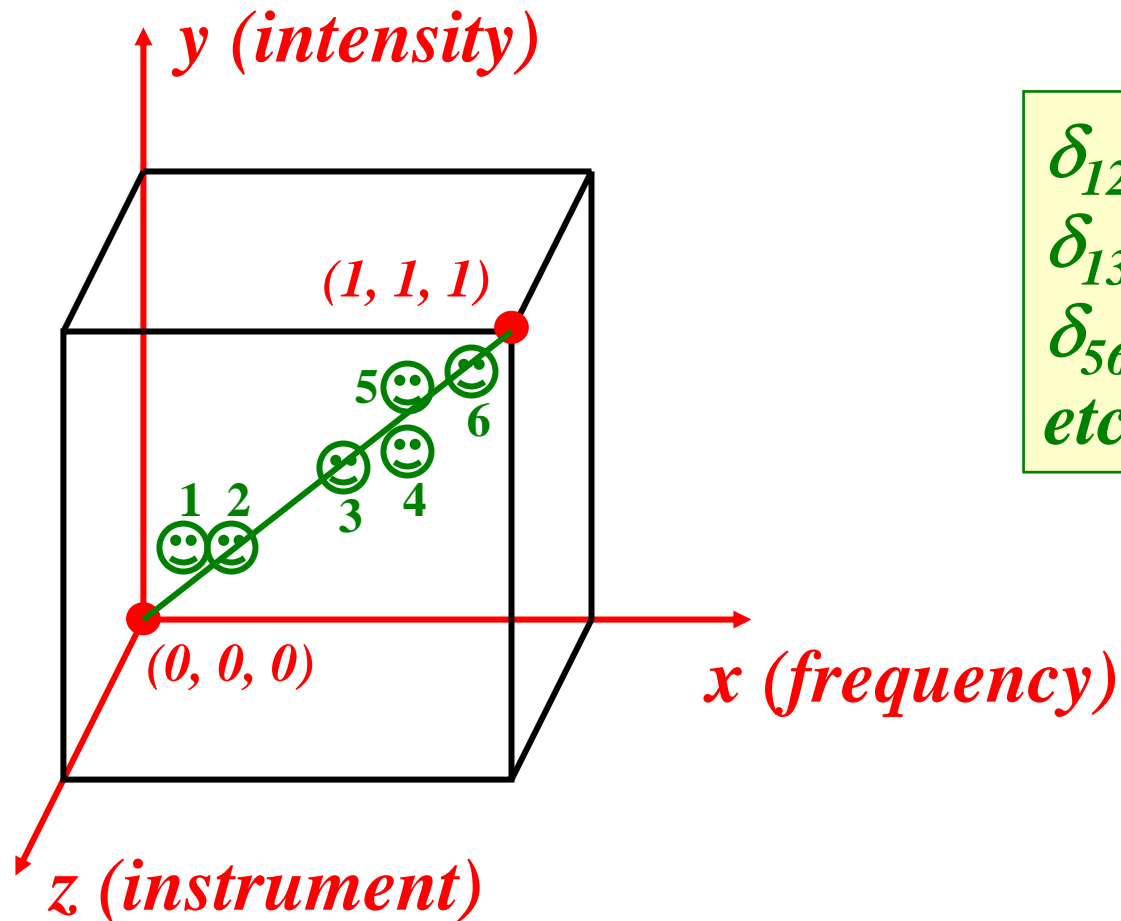
- In general, multidimensional stimuli lead to higher *information transfer*
- *Perceptual* dimensionality is related to, but not necessarily identical to, *physical* dimensionality
- Real-world stimuli are usually complex and multidimensional. How do we determine their associated *perceptual* dimensionality?
  - ◆ Example 1. Face recognition
  - ◆ Example 2. Color perception
  - ◆ Example 3. Haptic surface texture perception

# Multidimensional Scaling (MDS)

## — An Overview

- MDS is a technique that lets us investigate the underlying dimensionality associated with a stimulus set.
- Given a set of  $n$  objects
  - ◆ Obtain “dissimilarity” measures  $\delta_{rs}$  for each pair of objects  $(r, s)$
  - ◆ Search for a low dimensional perceptual space, where each object is represented by a point
  - ◆ Ensure that the distances between the points in perceptual space,  $\{d_{rs}\}$ , match the original dissimilarities  $\{\delta_{rs}\}$ .

# MDS — The Idea



# MDS — The Result(s)

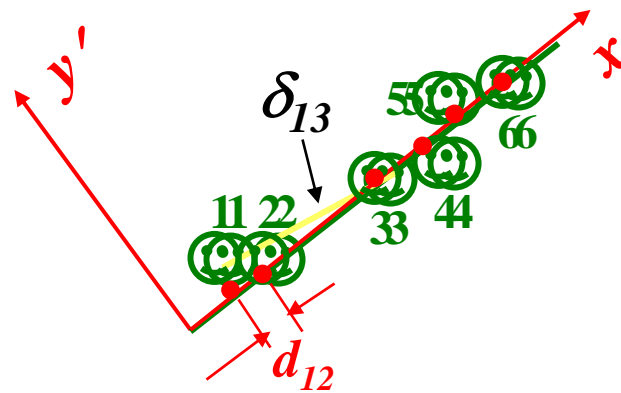
Dissimilarity  
Judgments

$$\delta_{12}=1.0$$

$$\delta_{13}=5.0$$

$$\delta_{56}=1.5$$

*etc.*



**Recovered  
Perceptual  
Distances**

# An Example: Texture Perception

- Hollins et al., *P&P*, 1993.
- Stimuli: 17 texture samples
- Procedure: passive stimulation
- Dissimilarity Scores
  - ◆ Grouping (i.e., similarity scores)
  - ◆ Co-occurrence scores (0.0 or 1.0)
  - ◆ Dissimilarity =  $1 - \text{Co-occurrence}$
- MDS analysis (ALSCAL, SAS)

# Co-occurrence Matrix

## Average over All Subjects

Stimulus	Felt	Straw	Wax Paper	Cork	Tile	Card-board
Felt	1.00					
Straw	.00	1.00				
Wax paper	.05	.00	1.00			
Cork	.10	.05	.30	1.00		
Tile	.05	.00	.60	.55	1.00	
Cardboard	.05	.00	.60	.60	.95	1.00

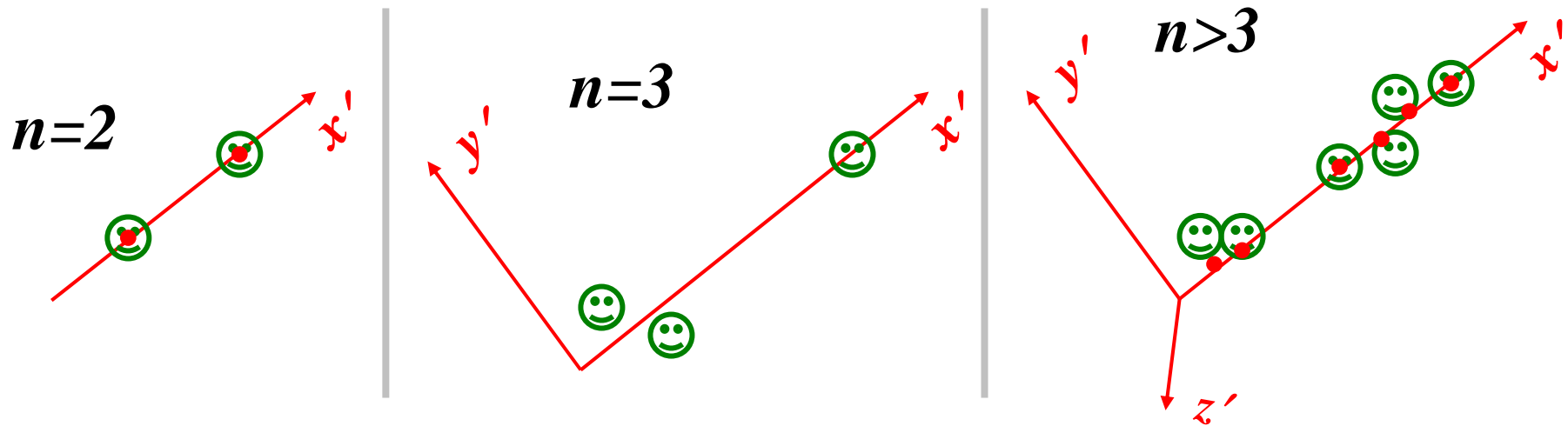


**(cont.)**

- **Dissimilarity = 1 – Co-occurrence**
- **MDS analysis (ALSCAL, SAS)**

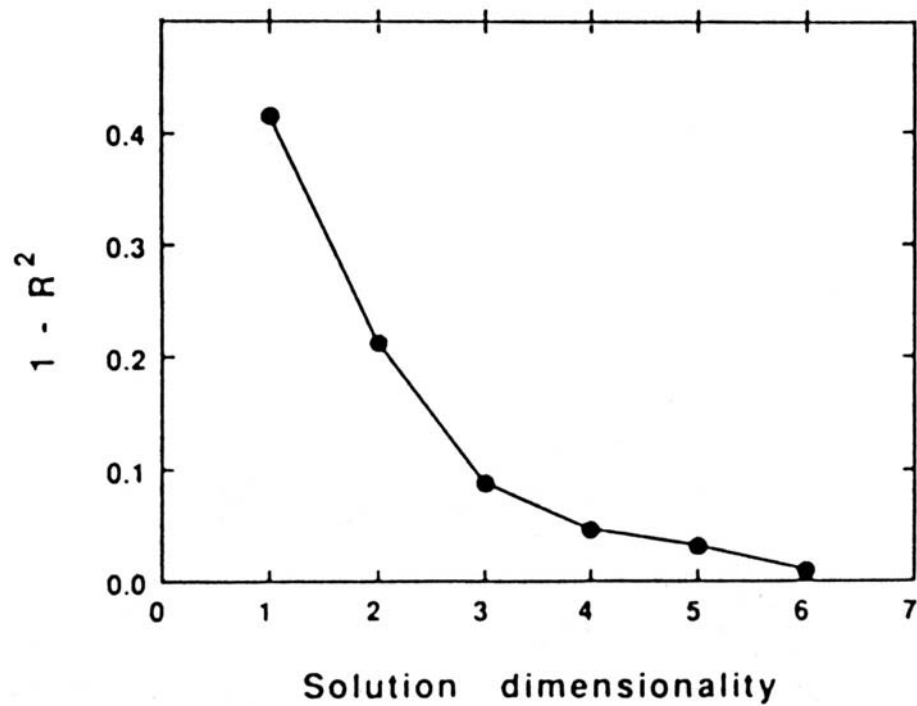
# How many dimensions?

- Given  $n$  objects, MDS analysis recovers  $(n-1)$  underlying dimensions

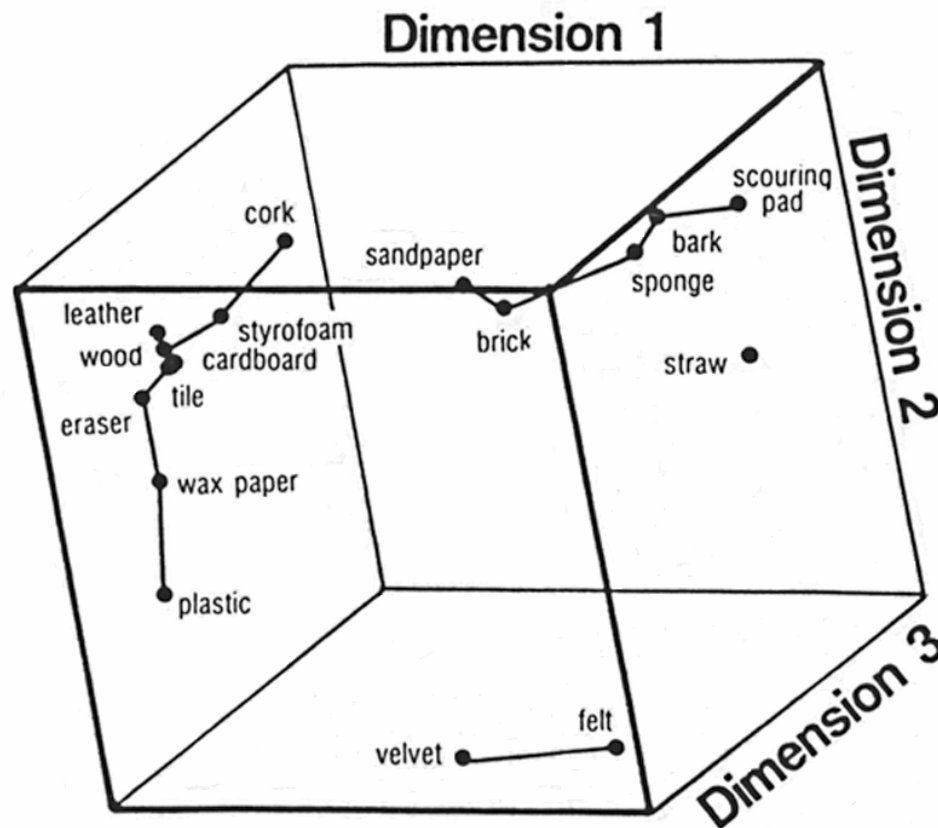


- Dimensionality is determined by examining S-Stress, Stress, and  $1-R^2$ , as a function of dimensions

# 1-R<sup>2</sup> Plot from Hollins *et al.*



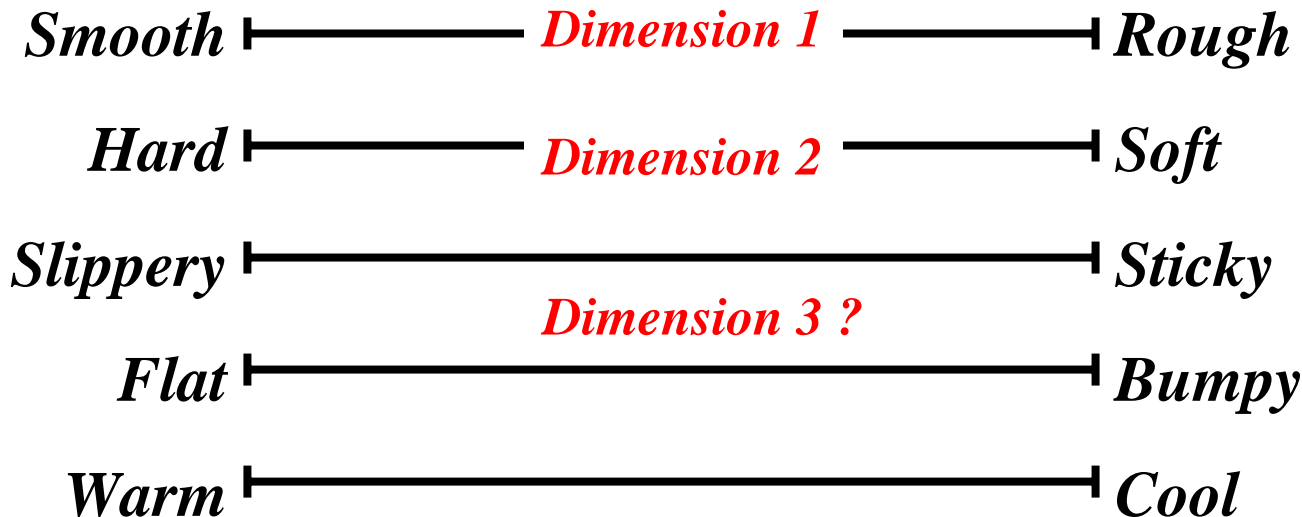
# A 3-D Solution



Cube representing the 3-dimensional multidimensional scaling solution.  
(From Hollins, Faldowski, Rao & Young, 1993)

# Interpreting the MDS Solution

## ■ Adjective Rating



# Summary of MDS

- **Experimental procedures**
  - ◆ **Key is to obtain dissimilarity scores**
  - ◆ **Grouping, similarity, dissimilarity**
  - ◆ **Ordering (non-metric)**
  - ◆ **etc.**
- **Data analysis**
  - ◆ **Use statistical packages such as SAS**
  - ◆ **For  $n$  objects,  $(n-1)$  dimensional solution**

- **Select solution dimensionality**
  - ◆ **S-Stress, Stress, and  $1 - R^2$**
- **Interpretation of MDS solutions**
  - ◆ **Adjective rating**
- **Known problems and limitations**
  - ◆ **Invariant to translation, rotation, reflection**
  - ◆ **May “discover” non-existent perceptual spaces**
- **Verification of MDS solution**
  - ◆ **Adjective rating**
  - ◆ **Matching experiments**
    - Color perception:  $X + rR = gG + bB$**

# Readings

- **T. F. Cox and M. A. A. Cox, *Multidimensional Scaling*. New York: Chapman & Hall, 1994.**
- **M. Hollins, R. Faldowski, S. Rao, and F. Young, “Perceptual dimensions of tactile surface texture: A multidimensional scaling analysis,” *Perception & Psychophysics*, vol. 54, pp. 697-705, 1993.**