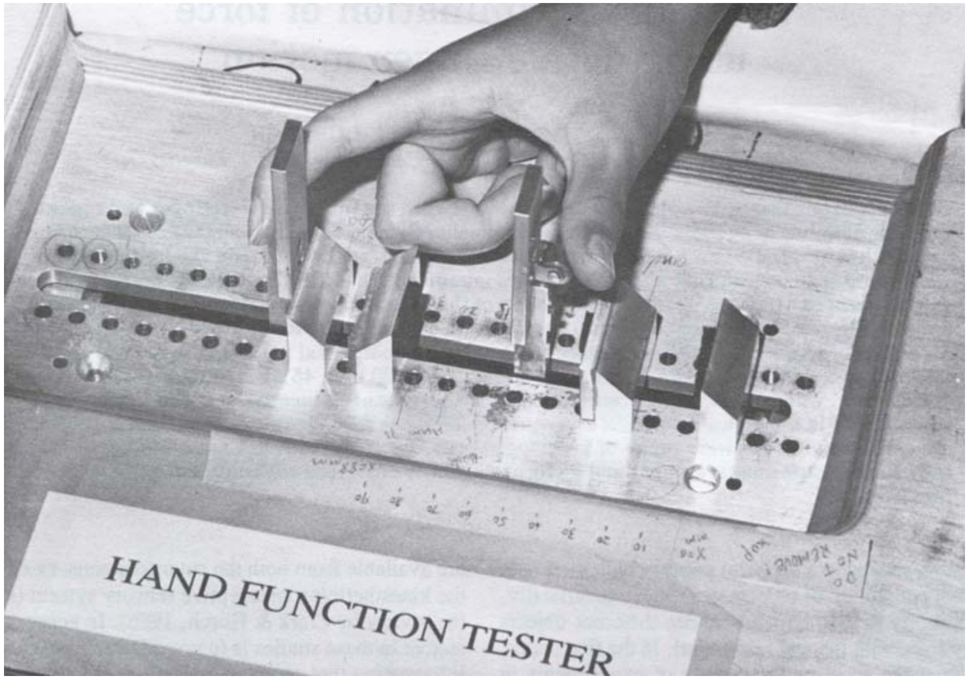
A Decision Model for Psychophysics (*Cont.*)

1. How to Test the Decision Model
2. Relating d′ to DL
3. Estimation of $\sigma_{d'}$

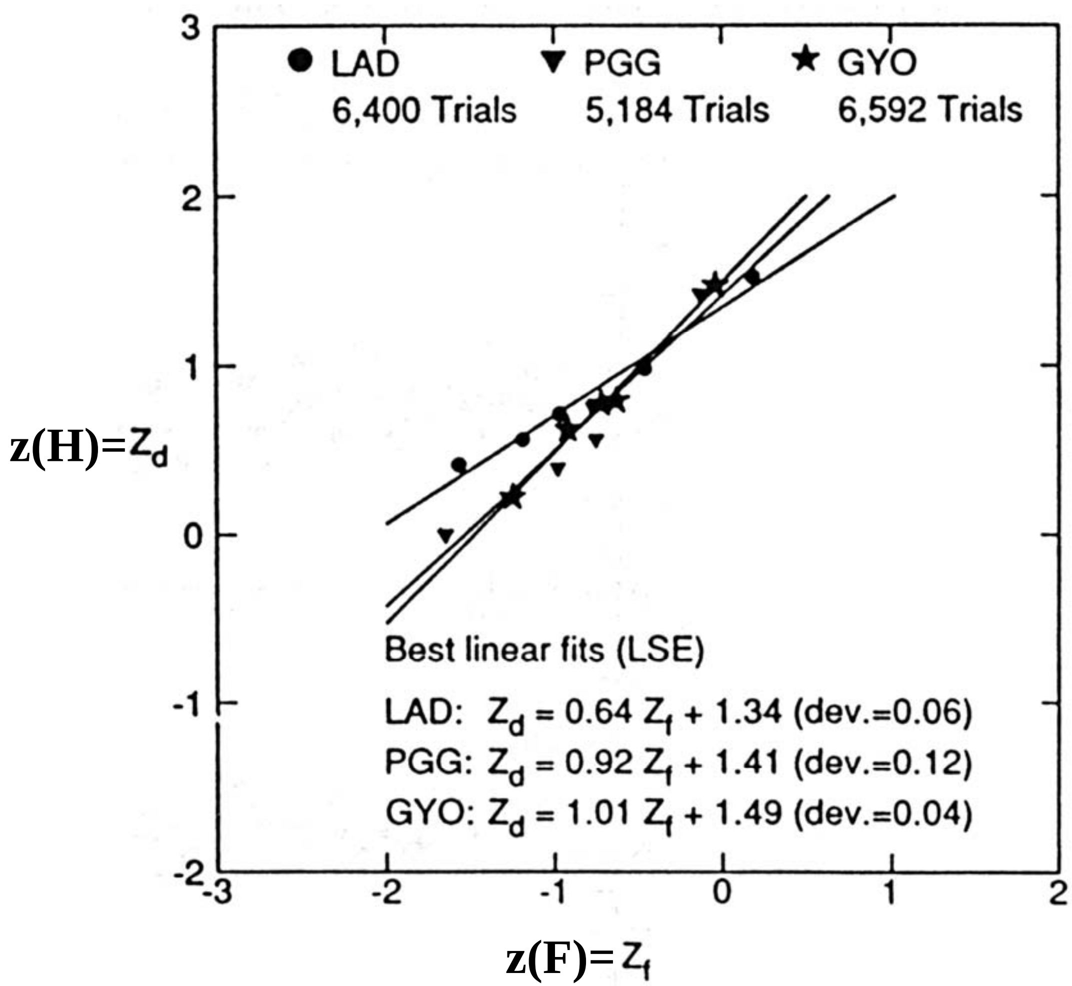
How to Test the Decision Model? — the Idea

- Pang, Tan and Durlach (*1991*)
- A study of manual force discrimination
- Q: Is ROC a straight line with slope of 1?
- Approach: Measure isosensitivity curve by using $P(S_1)$=1/10, 3/10, 5/10, 7/10 and 9/10.



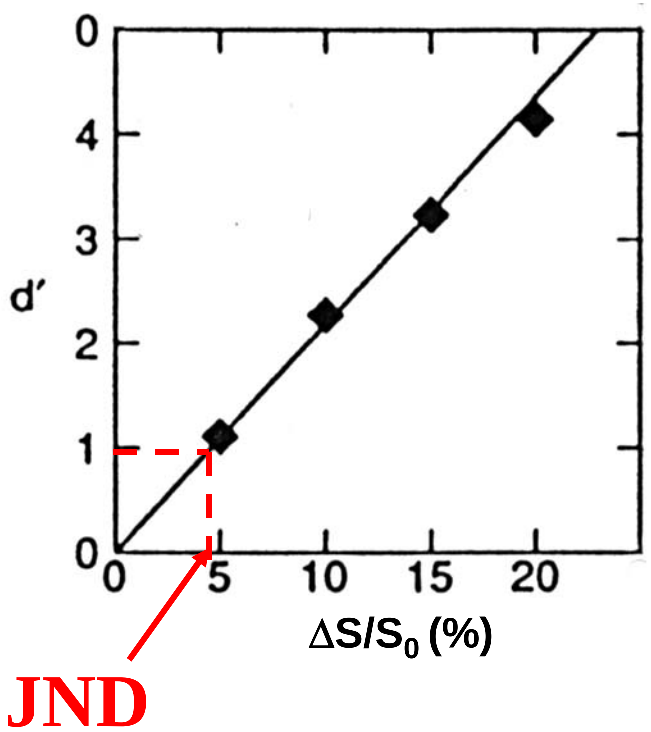
- Apparatus: A Linear Force Grasper
- Stimuli: S_1 = 5 N, S_2 = 5.5 N
- Responses: “smaller” or “larger” force

How to Test the Decision Model? — the Results



Difference Threshold *DL*, Sensitivity Index *d´*, Just Noticeable Difference *JND*, and their Relationship

JND



- **Just-Noticeable-Difference**
- **JND = AL (detection) or DL (discrimination)**
- **It is defined as**

$$\mathrm{JND} = 1/\bar{\delta}$$

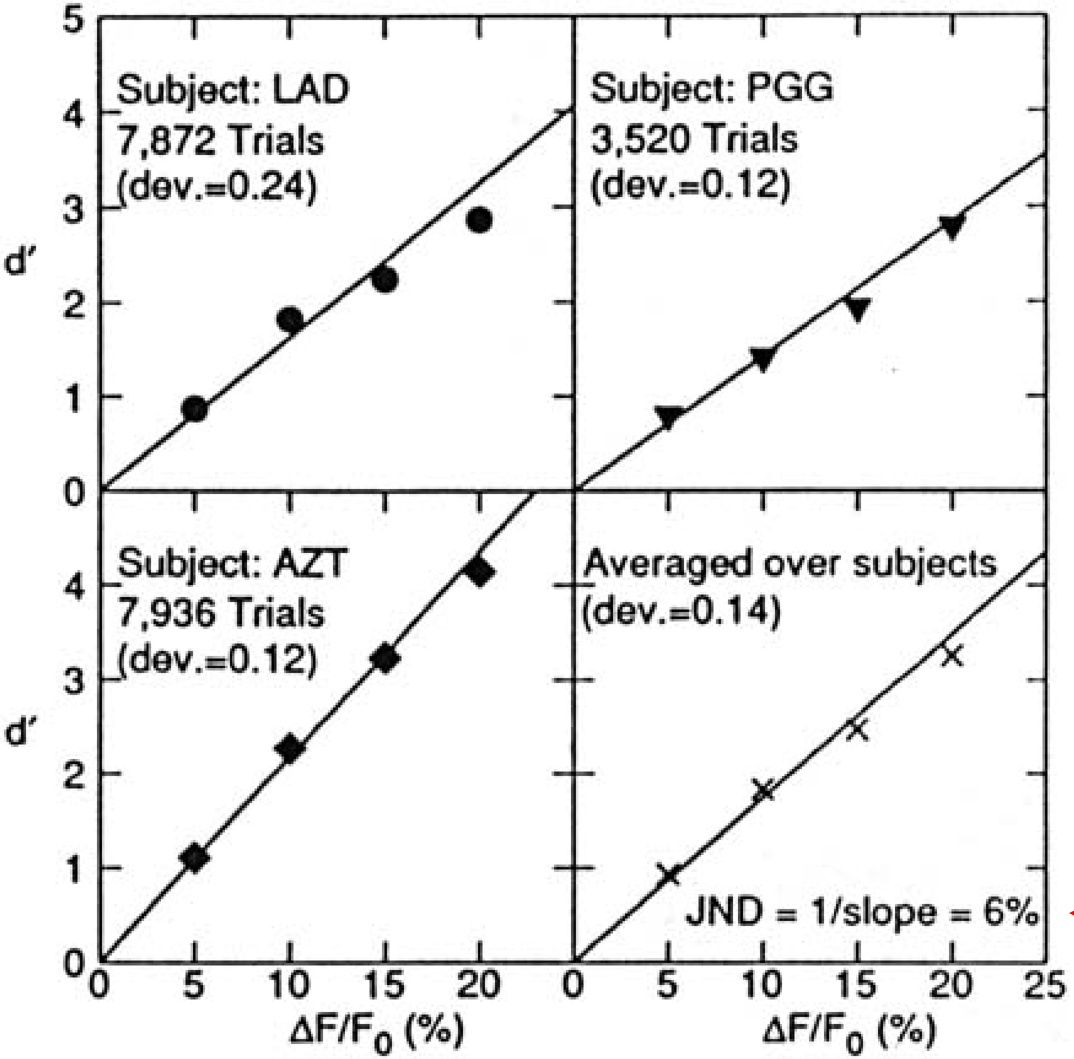
- $\bar{\delta}$ **is the average of d´/(ΔS/S_0) over several values of ΔS for a given S_0**
- **(S_1=S_0, S_2=S_0+ΔS)**

Estimating DL from d′

- From a 1-I experiment, we get d′
- In theory, DL can simply be computed as $DL = \Delta S / d'$
- However, estimating DL from one pair of (ΔS, d′) values is not reliable
- In practice, draw d′ vs. ΔS for many pairs of (ΔS, d′), then compute DL as the inverse of the $d' = \Delta S / DL$ slope.
- DL is the same as JND in a discrimination exp.
  - Be very careful with units (pixel, gram, vs. %)
  - Be aware of the reference signal level S_0

Real Data

From Pang, Tan & Durlach (1991)



← **JND**

Legend:

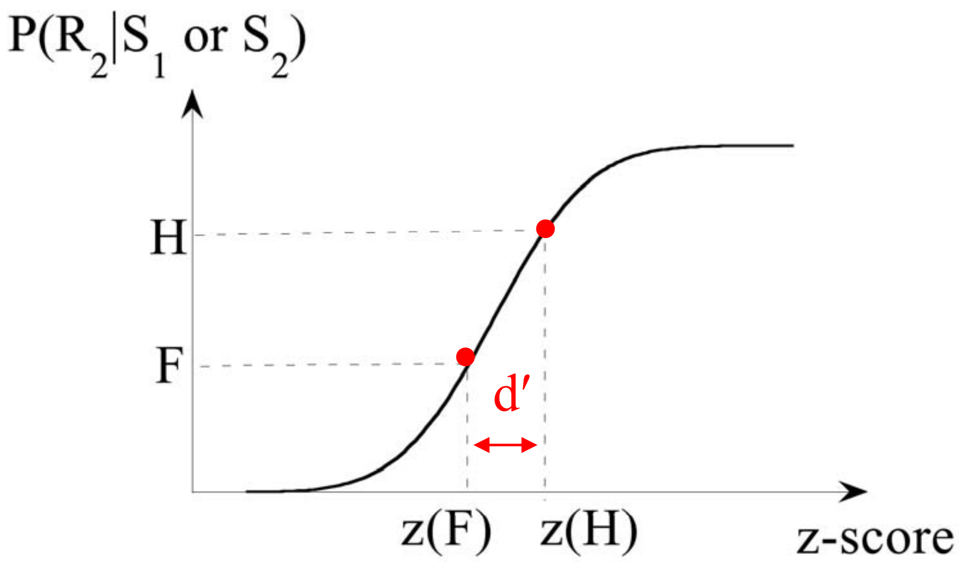
$S_1 = F_0 = 5$ N
$S_2 = F_0 + \Delta F$

$\Delta F/F_0 = \Delta S/S_1$

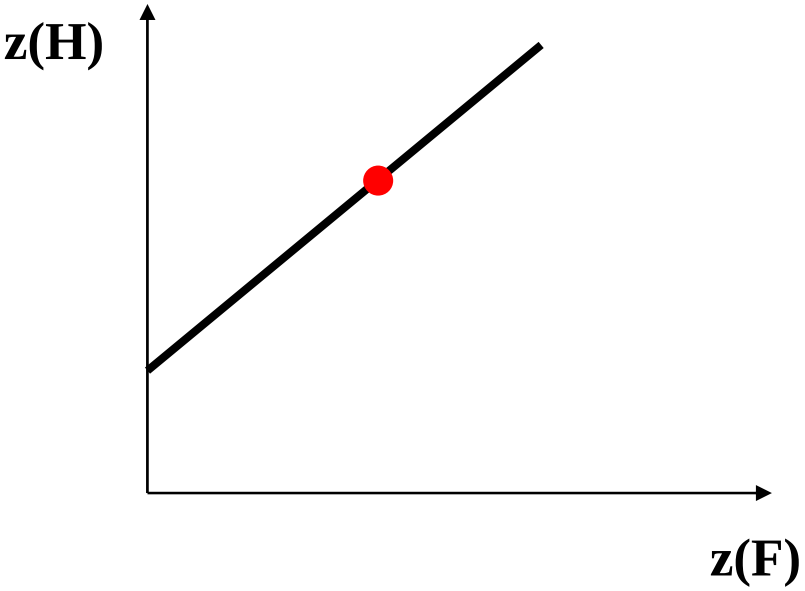
Estimation of $\sigma_{d'}$

The Problem

- $\sigma_{d'}$ can not be estimated directly from 1-I or 2-I experiments
- We can always find a cumulative Gaussian curve that goes through 2 points EXACTLY



- **We can always find a straight line that goes through 1 point EXACTLY**



- ***Therefore, we can NOT measure the goodness of fit in either case with rms error***

Two Ways to Estimate $\sigma_{d'}$

- 1. When you only collect one pair of (H, F)
  - Need assumptions to estimate $\sigma_{d'}$
- 2. When you measure multiple pairs of (H, F)
  - Use the rms error of ROC curve fitting
  - There are two ways to estimate ROC
    - Run multiple experiments. Ask subjects to use different *k* in different sessions.

      or
    - Rating paradigm. Ask subjects to maintain multiple *k* in the same session.

The Main Idea

- **Given that**

$$d' = z(H) - z(F)$$

- **We have**

$$\sigma_{d'} = \sqrt{\sigma^2_{z(H)} + \sigma^2_{z(F)}}$$

- **Therefore, to estimate $\sigma_{d'}$, we need to estimate $\sigma_{z(H)}$ and $\sigma_{z(F)}$ first.**

Method 1: Estimate $\sigma_{d'}$ with One Pair of (H, F) Values

- **Assumptions**
  - **The only source of variability in d′ is sampling error (therefore we are getting a lower-bound estimate)**
  - **Binomial distribution approximates Gaussian with sufficient number of trials. This is true if**
    - ***p* (probability of responding "yes") is not extreme; or**
    - ***N* (number of trials) is large when *p* is extreme**

- **It then follows that the variability of H and F are:**

$$\sigma_F = \sqrt{\frac{F(1-F)}{N_1}}$$

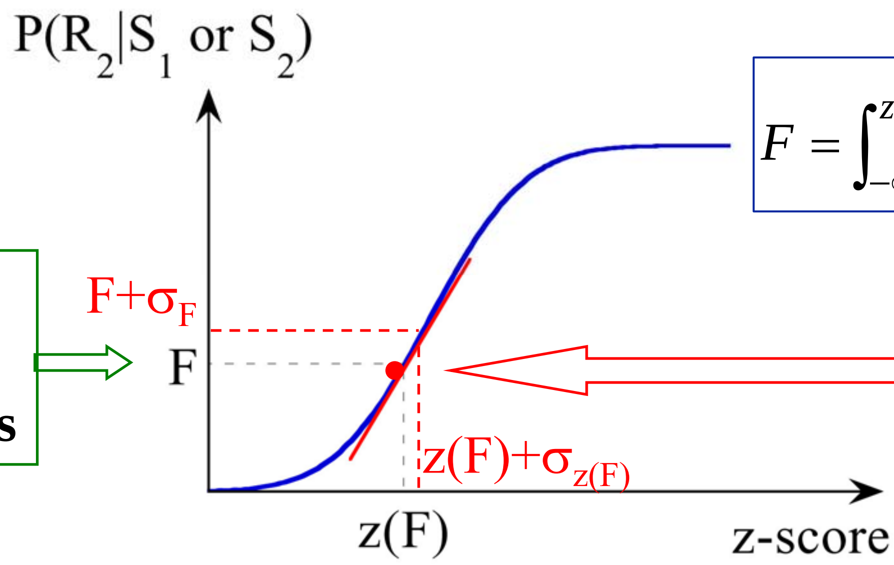
$$\sigma_H = \sqrt{\frac{H(1-H)}{N_2}}$$

where N_1 or N_2 is the number of times stimulus S_1 or S_2 has been presented, respectively.

- **All we need to do now is to estimate $\sigma_{z(H)}$ and $\sigma_{z(F)}$.**

$$F = \int_{-\infty}^{z(F)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\, dx$$

N(F, σ^2_F) from assumptions



$F = \alpha \cdot z(F) + \beta$
(α,β: constants)
$\sigma_F = \alpha \cdot \sigma_{z(F)}$

N[z(F), $\sigma^2_{z(F)}$] based on local linear approximation

$$\alpha = \frac{\partial F}{\partial z(F)} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}[z(F)]^2}$$

$$\therefore \sigma_{z(F)} = \alpha^{-1} \cdot \sigma_F = \sigma_F \cdot \sqrt{2\pi} \cdot e^{\frac{1}{2}[z(F)]^2}$$

Summary of Method 1

$$\sigma_{z(F)} = \sigma_F \cdot \sqrt{2\pi} \cdot e^{\frac{1}{2}[z(F)]^2} = \sqrt{\frac{2\pi F(1-F)}{N_1}} \cdot e^{\frac{1}{2}[z(F)]^2}$$

$$\sigma_{z(H)} = \sqrt{\frac{2\pi H(1-H)}{N_2}} \cdot e^{\frac{1}{2}[z(H)]^2}$$

$$\text{Finally, } \sigma_{d'} = \sqrt{\sigma_{z(H)}^2 + \sigma_{z(F)}^2}$$

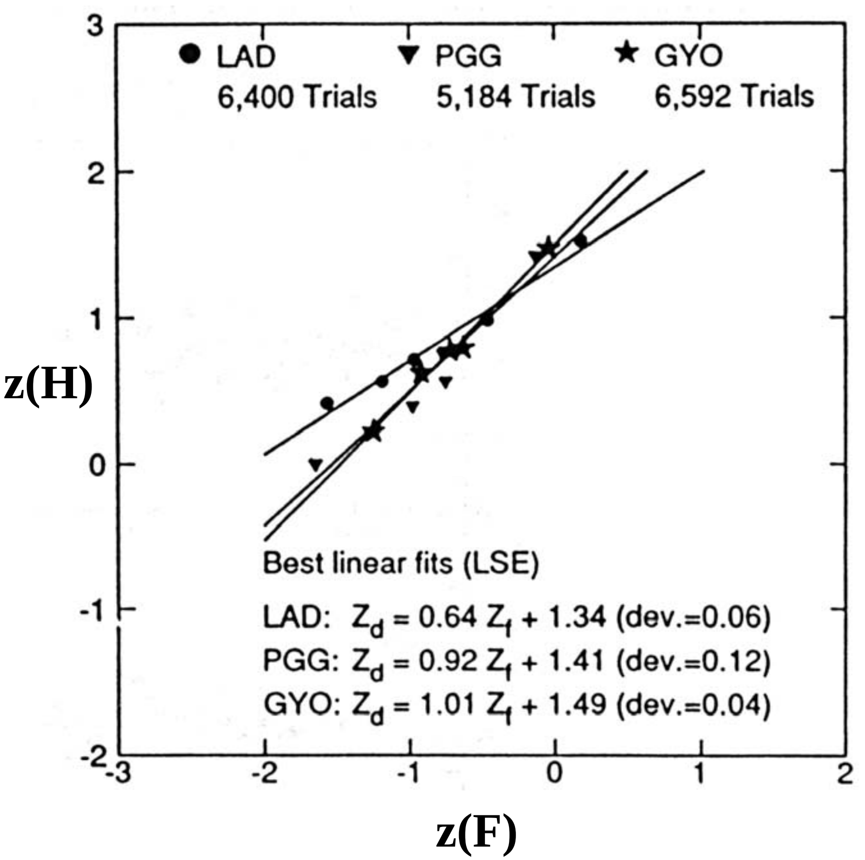
It should be clear that $\sigma_{d'}$ is inversely proportional to N_1 and N_2!

Method 2: Estimate $\sigma_{d'}$ with ROC

- No explicit assumptions are needed
- Estimate $\sigma_{z(H)}$ and $\sigma_{z(F)}$ as the rms error of straight line fitting

  How??
- Then compute

$$\sigma_{d'} = \sqrt{\sigma^2_{z(H)} + \sigma^2_{z(F)}}$$



Two Ways of Obtaining ROC

- **Multiple sessions with different *k* values**
  - **see Pang et al. 1991**
- **Same session with multiple *k* values**
  - **Rating Experiment**

Reading

- **Pang, Tan and Durlach, "Manual discrimination of force using active finger motion,"** ***Perception & Psychophysics*****, 49(6), 531–540, 1991.**