Data Analysis for an Absolute Identification Experiment

Randomization with Replacement

- Imagine that you have *k* containers for the *k* stimulus alternatives
- The i_{th} container has a fixed number of copies (n_i, proportional to $P(S_i)$) of the i_{th} stimulus
- On each trial, one of the Σn_i (i=1, …, k) stimuli is selected to be presented to the subject
- That stimulus is immediately replaced in its corresponding container
- Then, the *a priori* probability for S_i (i=1, …, k) remains the same for all trials
- The stimulus uncertainty remains the same on all trials

$$IS = -\sum_{i=1}^{k} P(S_i)\log_2 P(S_i)$$

Randomization without Replacement

- Imagine that you have *k* containers for the *k* stimulus alternatives
- The i_{th} container has a fixed number of copies (n_i, proportional to $P(S_i)$) of the i_{th} stimulus
- On each trial, one of the Σn_i (i=1, …, k) stimuli was selected to be presented to the subject
- That stimulus is NOT replaced in its corresponding container
- Then, the *a priori* probability for S_i may change from trial to trial
- The stimulus uncertainty *IS* may change from trial to trial
- On the last trial, the subject knows exactly what stimulus to expect (whichever stimulus is the last one left in a container)

More on Randomization

- We prefer the method of "randomization with replacement" because
  - It ensures constant *IS* for each trial
  - It makes data analysis easier
- With the method of "randomization with replacement," equal *a priori* probability no longer guarantees equal number of occurrences for all stimulus alternatives.
- Note that frequency of occurrence ≠ probability
- The advantage of "randomization without replacement" is that the experimenter controls the *exact* number of times each stimulus alternatives is presented.

	R_1	R_2	R_3	R_4	R_5	
S_1	14	3	2	0	1	20
S_2	0	13	2	3	1	19
S_3	4	3	11	1	0	19
S_4	2	0	2	15	1	20
S_5	5	3	2	0	12	22
	25	22	19	19	15	100

Estimation of IT — IT_{est}

- **Average information transfer:**

$$IT = \sum_{j=1}^{k}\sum_{i=1}^{k} P(S_i, R_j)\log_2\frac{P(S_i|R_j)}{P(S_i)}$$

- **Its maximum-likelihood estimate:**

$$IT_{est} = \sum_{j=1}^{k}\sum_{i=1}^{k}\left(\frac{n_{ij}}{n}\right)\log_2\left(\frac{n_{ij}\cdot n}{n_i \cdot n_j}\right)$$

where n_{ij}

$$n_i = \sum_{j=1}^{k} n_{ij} \qquad n_j = \sum_{i=1}^{k} n_{ij}$$

$$n = \sum_{j=1}^{k}\sum_{i=1}^{k} n_{ij} = \sum_{i=1}^{k} n_i = \sum_{j=1}^{k} n_j$$

- **Interpretation of 2^{IT} or $2^{IT_{est}}$ (compare with $k=2^U$)**

Percent-correct scores and IT_{est}

$$IT_{est} = \sum_{j=1}^{k}\sum_{i=1}^{k}\left(\frac{n_{ij}}{n}\right)\log_2\left(\frac{n_{ij}\cdot n}{n_i\cdot n_j}\right)$$

(A)

25	25
25	25

50%
0 *bits*

(B)

25	25	25	25
25	25	25	25
25	25	25	25
25	25	25	25

25%
0 *bits*

(C)

25	0	0	0
0	25	0	0
0	0	25	0
0	0	0	25

100%
2 *bits*

(D)

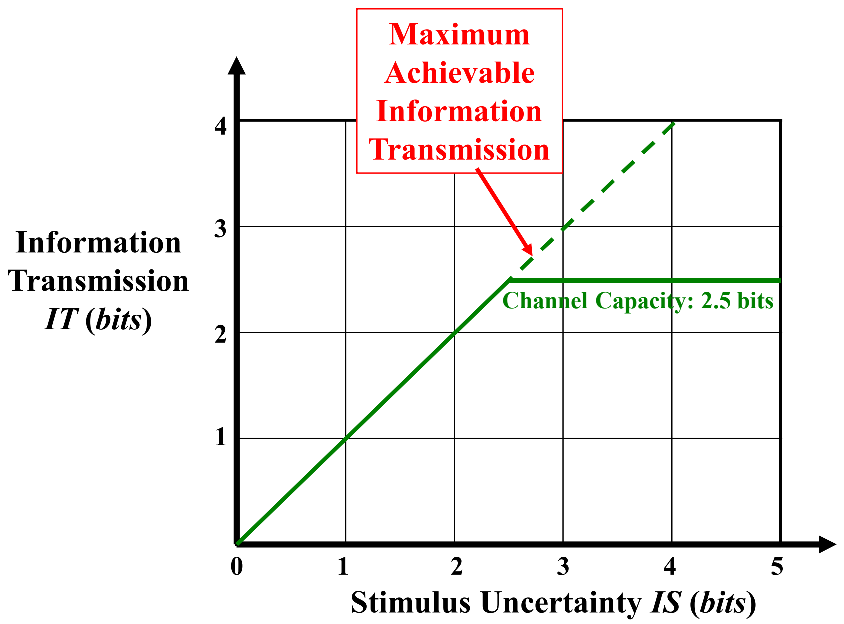
0	0	0	25
0	0	25	0
0	25	0	0
25	0	0	0

0%
2 *bits*

Channel Capacity

Maximum Information Transmission

- Mathematically, $IT \leq IS$.
- Intuitively, if the input and output are perfectly correlated, then $IT = IS$ (= IR).
- Assume that there exists a *maximum* information transmission
  - For small values of IS, $IT = IS$.
  - As IS increases, IT = *constant* regardless of the value of IS.
- This maximum IT is accepted as the ***channel capacity***.



The Magic Number 7±2

What does the "Magic Number" Mean?

- **The "magic number" is derived from an *IT* range of 2.3 – 3.2 *bits***
- **The "magic number" summarizes the typical *channel capacity* for uni-dimensional stimuli**
- **Uni-dimensional stimuli**
  - **Only one physical variables (*target*) is manipulated to form the stimulus set**
  - **Other physical variables (*background*) are either held constant or randomized**

How "Magic" is the Magic Number?

- The "Magic Number" does NOT apply to
  - Absolute pitch
    - Over-learnt stimuli
  - Human face recognition
    - Multi-dimensional stimuli

Reading

- G. A. Miller, "The magical number seven, plus or minus two: Some limits on our capacity for processing information," *The Psychological Review*, vol. 63, pp. 81-97, 1956.