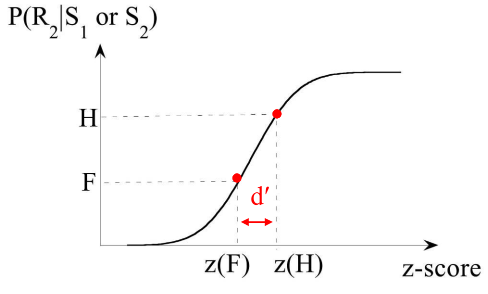
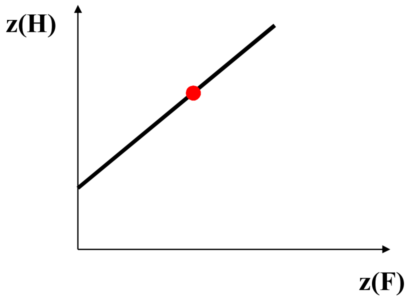
Estimation of $\sigma_{d'}$

The Problem

- $\sigma_{d'}$ can not be estimated directly from 1-I or 2-I experiments
- We can always find a cumulative Gaussian curve that goes through 2 points EXACTLY



- We can always find a straight line that goes through 1 point EXACTLY



- *Therefore, we can NOT measure the goodness of fit in either case with rms error*

Two Ways to Estimate $\sigma_{d'}$

- 1. When you only collect one pair of (H, F)
  - Need assumptions to estimate $\sigma_{d'}$
- 2. When you measure multiple pairs of (H, F)
  - Use the rms error of ROC curve fitting
  - There are two ways to estimate ROC
    - Run multiple experiments. Ask subjects to use different *k* in different sessions.

      or
    - Rating paradigm. Ask subjects to maintain multiple *k* in the same session.

The Main Idea

- Given that

$$d' = z(H) - z(F)$$

- We have

$$\sigma_{d'} = \sqrt{\sigma^2_{z(H)} + \sigma^2_{z(F)}}$$

- Therefore, to estimate $\sigma_{d'}$, we need to estimate $\sigma_{z(H)}$ and $\sigma_{z(F)}$ first.

Method 1: Estimate $\sigma_{d'}$ with One Pair of (H, F) Values

- Assumptions
  - The only source of variability in d′ is sampling error (therefore we are getting a lower-bound estimate)
  - Binomial distribution approximates Gaussian with sufficient number of trials. This is true if
    - *p* (probability of responding "yes") is not extreme; or
    - *N* (number of trials) is large when *p* is extreme

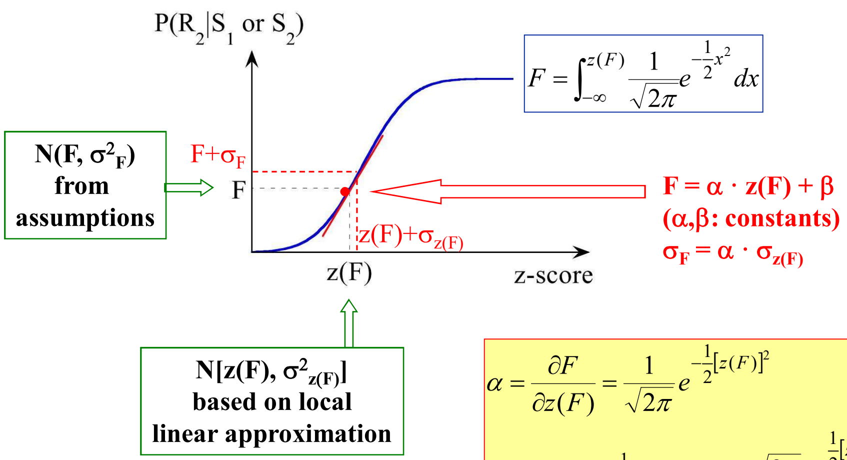
- **It then follows that the variability of H and F are:**

$$\sigma_F = \sqrt{\frac{F(1-F)}{N_1}}$$

$$\sigma_H = \sqrt{\frac{H(1-H)}{N_2}}$$

where N_1 or N_2 is the number of times stimulus S_1 or S_2 has been presented, respectively.

- **All we need to do now is to estimate $\sigma_{z(H)}$ and $\sigma_{z(F)}$.**



$$\alpha = \frac{\partial F}{\partial z(F)} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}[z(F)]^2}$$

$$\therefore \sigma_{z(F)} = \alpha^{-1} \cdot \sigma_F = \sigma_F \cdot \sqrt{2\pi} \cdot e^{\frac{1}{2}[z(F)]^2}$$

Summary of Method 1

$$\sigma_{z(F)} = \sigma_F \cdot \sqrt{2\pi} \cdot e^{\frac{1}{2}[z(F)]^2} = \sqrt{\frac{2\pi F(1-F)}{N_1}} \cdot e^{\frac{1}{2}[z(F)]^2}$$

$$\sigma_{z(H)} = \sqrt{\frac{2\pi H(1-H)}{N_2}} \cdot e^{\frac{1}{2}[z(H)]^2}$$

$$\text{Finally, } \sigma_{d'} = \sqrt{\sigma^2_{z(H)} + \sigma^2_{z(F)}}$$

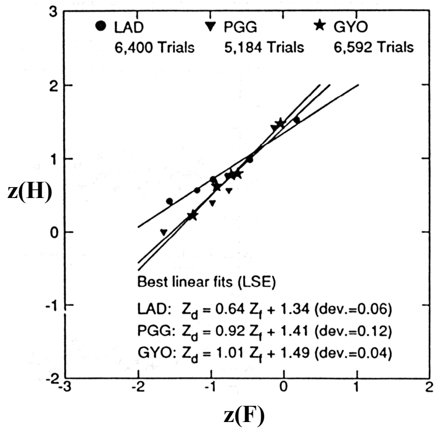
It should be clear that $\sigma_{d'}$ is inversely proportional to N_1 and N_2!

Method 2: Estimate $\sigma_{d'}$ with ROC

- **No explicit assumptions are needed**
- **Estimate $\sigma_{z(H)}$ and $\sigma_{z(F)}$ as the rms error of straight line fitting**

  How??
- **Then compute**

  $$\sigma_{d'} = \sqrt{\sigma^2_{z(H)} + \sigma^2_{z(F)}}$$



Two Ways of Obtaining ROC

- Multiple sessions with different *k* values
  - see Pang et al. 1991
- Same session with multiple *k* values
  - Rating Experiment