

Axiom of Extensionality

$$(\forall x) (x \in S \iff x \in T)$$

$\updownarrow$

$S = T$

$$(\forall x) (x \in S \Rightarrow x \in T)$$

$$S \subseteq T$$

$$T \subseteq S \quad \leftarrow$$

$$S \subseteq T \wedge T \subseteq S \rightarrow S = T$$

$$\alpha \quad \lambda x. e_1 = \lambda y. e_1[x \rightsquigarrow y]$$



alpha
alpha renaming
conversion

$$\beta \quad (\lambda x. e_1)(e_2) = e_1[x \rightsquigarrow e_2]$$

$\begin{matrix} \rightarrow \\ \leftarrow \end{matrix}$

β reduction
 β expansion

$\eta \quad \lambda x. f(x) = f$

$\rightarrow \eta$ reduction

$\leftarrow \eta$ expansion

$E ::= \checkmark$

variable reference

$| e_1(e_2)$

function call

$| \lambda v. e$

function definition

2 0 $\mathbb{Z}^+$

0
0 $\mathbb{N}$

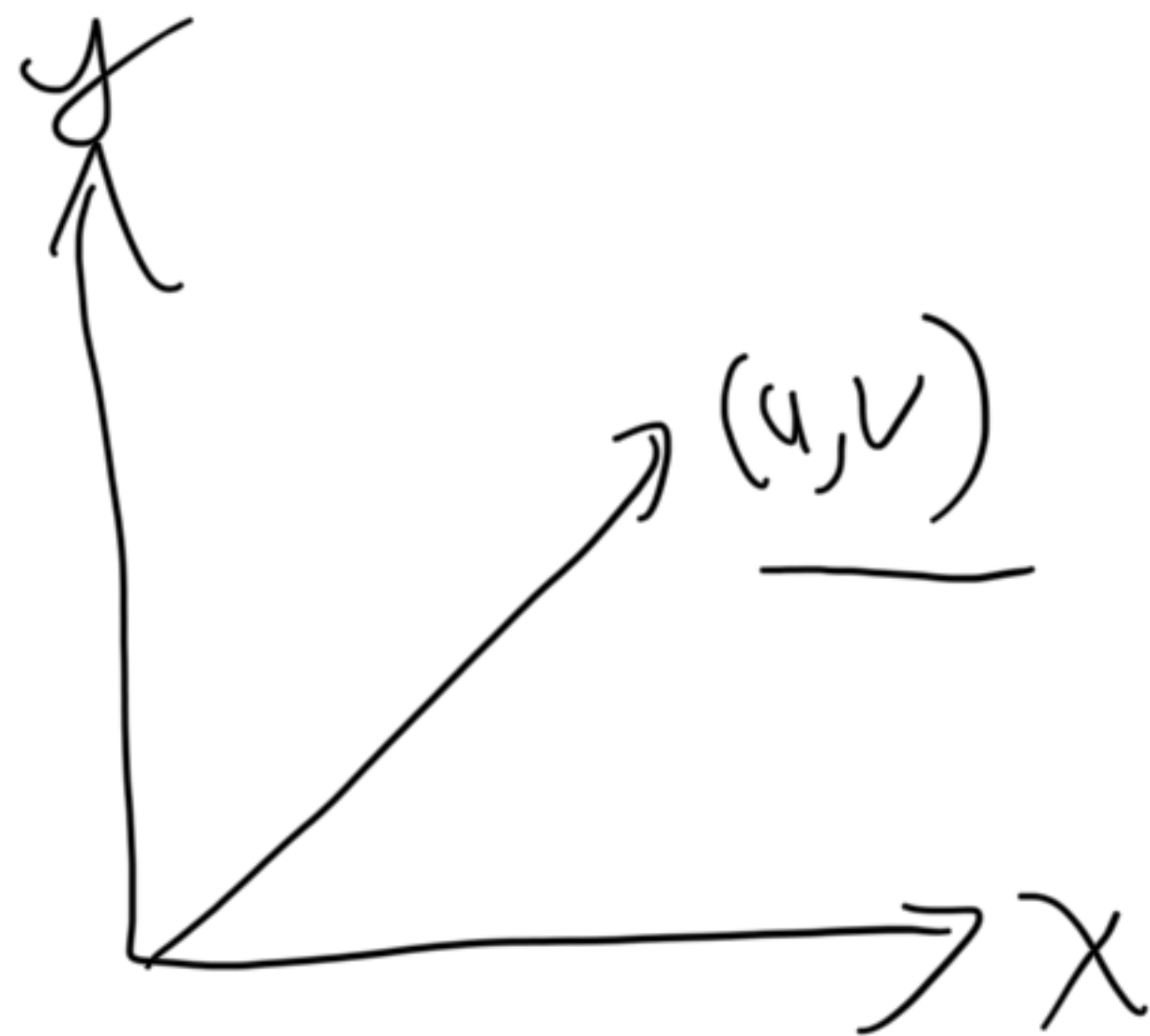
π 0 $\mathbb{R}$

$2/3$ 0
0 $\mathbb{Q}$

$2+3i$ 0
0 $\mathbb{C}$

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$$





Cartesian Coordinates

de Cartes

$\mathbb{R}^2$

$\mathbb{R}^{m \times n}$

$\mathbb{R}^3$

$\mathbb{R}^n$

$$\sin(x)$$

$$\sin : \mathbb{R} \rightarrow \mathbb{R}$$

$$+ : \mathbb{R}^2 \rightarrow \mathbb{R} \quad (\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R})$$

$$\mathbb{R} \rightarrow (\mathbb{R} \rightarrow \mathbb{R})$$

$$\frac{d}{dx} : (\mathbb{R} \rightarrow \mathbb{R}) \rightarrow (\mathbb{R} \rightarrow \mathbb{R})$$

$$\nabla : (\mathbb{R}^n \rightarrow \mathbb{R}) \rightarrow (\mathbb{R}^n \rightarrow \mathbb{R}^n)$$

$$\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$+ : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$

$$((\mathbb{R} \rightarrow \mathbb{R}) \times (\mathbb{R} \rightarrow \mathbb{R})) \rightarrow (\mathbb{R} \rightarrow \mathbb{R})$$

$$f + g \stackrel{\Delta}{=} \lambda x. (f(x) + g(x))$$

pointwise lifting