# ECE49595NL Lecture 18: Montague Grammar—I

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# First-Order Logic—I

A method for formally representing the meaning of some English sentences

John is a professor.

$$professor(John)$$

John ate breakfast.

$$ate(John, breakfast)$$

John ate cereal and drank juice.

$$ate(John, cereal) \wedge drink(John, juice)$$

John didn't eat toast.

$$\neg eat(John, toast)$$

If John leaves the milk outside the refrigerator it will spoil.

$$leave(John, milk, outside\text{-}refrigerator) \rightarrow will\text{-}spoil(milk)$$

# First-Order Logic—II

All pawns are pieces.

$$\forall x(\mathit{pawn}(x) \rightarrow \mathit{piece}(x))$$

Some squares are corners.

$$\exists x(\mathit{square}(x) \land \mathit{corners}(x))$$

All white pawns are pieces.

$$\forall x((\mathit{white}(x) \land \mathit{pawn}(x)) \rightarrow \mathit{piece}(x))$$

Some black squares are corners.

$$\exists x(\mathit{black}(x) \land \mathit{square}(x) \land \mathit{corner}(x))$$

# First-Order Logic—III

Some pawns are on some squares.

$$\exists x(\mathit{pawn}(x) \land \exists y(\mathit{square}(y) \land \mathit{on}(x, y)))$$

Every piece is used in every game.

$$\forall x(\mathit{piece}(x) \rightarrow \forall y(\mathit{game}(y) \rightarrow \mathit{used\text{-}in}(x, y)))$$

Every piece is on some square.

$$\forall x(\mathit{piece}(x) \rightarrow \exists y(\mathit{square}(y) \land \mathit{on}(x, y)))$$

$$\exists y(\mathit{square}(y) \land \forall x(\mathit{piece}(x) \rightarrow \mathit{on}(x, y)))$$

Some piece is on every square.

$$\exists x(\mathit{piece}(x) \land \forall y(\mathit{square}(y) \rightarrow \mathit{on}(x, y)))$$

$$\forall y(\mathit{square}(y) \rightarrow \exists x(\mathit{piece}(x) \land \mathit{on}(x, y)))$$

# The Lambda Calculus

```
float distance (float x1, float y1, float x2, float y2)
return sqrt((x2-x1)**2+(y2-y1)**2);

z = distance(a+b, c+d, e+f, g+h);

z = sqrt(((e+f)-(a+b))**2+((g+h)-(c+d))**2)
```

$$distance = \lambda(x_1, y_1, x_2, y_2)\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\begin{aligned}
&distance(a+b, c+d, e+f, g+h)\\
&\quad = \lambda(x_1, y_1, x_2, y_2)\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}(a+b, c+d, e+f, g+h)\\
&\quad = \sqrt{((e+f)-(a+b))^2+((g+h)-(c+d))^2}
\end{aligned}$$

$$distance = \lambda x_1 \lambda y_1 \lambda x_2 \lambda y_2 \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\begin{aligned}
&(((distance(a+b))(c+d))(e+f))(g+h)\\
&\quad = ((((\lambda x_1 \lambda y_1 \lambda x_2 \lambda y_2 \sqrt{(x_2-x_1)^2+(y_2-y_1)^2})(a+b))(c+d))(e+f))(g+h)\\
&\quad = (((\lambda y_1 \lambda x_2 \lambda y_2 \sqrt{(x_2-(a+b))^2+(y_2-y_1)^2})(c+d))(e+f))(g+h)\\
&\quad = ((\lambda x_2 \lambda y_2 \sqrt{(x_2-(a+b))^2+(y_2-(c+d))^2})(e+f))(g+h)\\
&\quad = (\lambda y_2 \sqrt{((e+f)-(a+b))^2+(y_2-(c+d))^2})(g+h)\\
&\quad = \sqrt{((e+f)-(a+b))^2+((g+h)-(c+d))^2}
\end{aligned}$$

# Montague Grammar

*Some pawn is black.*

$$
\begin{aligned}
\textit{Some} &= \lambda x \lambda y \exists z((xz) \wedge (yz)) \\
\textit{pawn} &= \lambda u(\textit{pawn}(u)) \\
\textit{is} &= \lambda v(v) \\
\textit{black} &= \lambda w(\textit{black}(w))
\end{aligned}
$$

$$\exists z(\textit{pawn}(z) \wedge \textit{black}(z))$$
$$\exists z(\textit{pawn}(z) \wedge ((\lambda w(\textit{black}(w)))z))$$
$$(\lambda y \exists z(\textit{pawn}(z) \wedge (yz)))(\lambda w(\textit{black}(w)))$$

$$\lambda y \exists z(\textit{pawn}(z) \wedge (yz))$$
$$\lambda y \exists z(((\lambda u(\textit{pawn}(u)))z) \wedge (yz))$$
$$(\lambda x \lambda y \exists z((xz) \wedge (yz)))(\lambda u(\textit{pawn}(u)))$$

*Some* *pawn*

$$\lambda w(\textit{black}(w))$$
$$(\lambda v(v))(\lambda w(\textit{black}(w)))$$

*is* *black*