EE 438 Digital Signal Processing with Applications: Classification of Signals According to their Support

Prof. Jan P. Allebach
School of Electrical and Computer Engineering
Purdue University
West Lafayette IN 47907-1285
allebach@ecn.purdue.edu

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1 Introduction

In this note, we categorize signals according to where they are non-zero on the time axis. The set of points where a signal is non-zero is called its *support*. This classification could be done in many different ways. Here we choose a categorization that corresponds to the convergence properties of the Z transform of the signal, which we will be discussing later in the course. We consider explicitly only the discrete-time case. Analogous definitions can be made for the continuous-time case, as well. In this case, we would be considering convergence properties of the Laplace transform.

2 Finite vs. Infinite Duration Signals

First, we divide all signals into two classes: those that are of finite duration and those that are of infinite duration. A signal $x[n]$ is of *finite duration* if there exists two integers $-\infty < N_1 \le N_2 < \infty$, such that $x[n] \ne 0$ only for $N_1 \le n \le N_2$. Otherwise, it is of *infinite duration*.

3 Right-sided, Left-Sided, and Two-Sided Signals

These terms apply only to signals that are of infinite duration. We say that a signal $x[n]$ is *right-sided* if there exists an integer N_1 such that $x[n] \ne 0$ only for $N_1 \le n$. The signal $x[n]$ is *left-sided* if there exists an integer N_2 such that $x[n] \ne 0$ only for $n \le N_2$. If $x[n]$ is of infinite duration and it is neither right-sided nor left-sided, then it is *two-sided*.
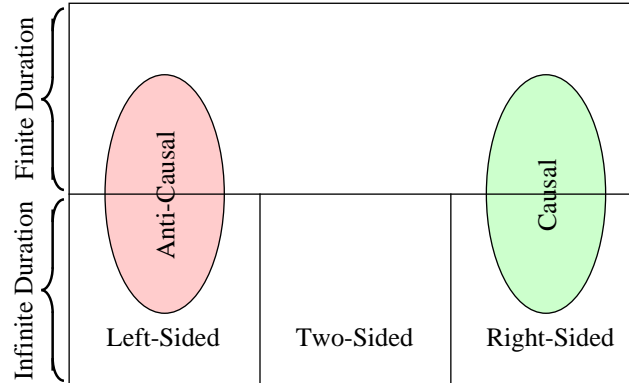
Figure 1: Classification of signal support.

4   Causal and Anti-Causal Signals

These terms apply to both finite and infinite duration signals. A signal $x[n]$ is *causal* if $x[n] = 0$ for all $n < 0$. It is *anti-causal* if $x[n] = 0$ for all $n > 0$. It follows that if $x[n]$ is of infinite duration, then causality implies right-sidedness and anti-causality implies left-sidedness.

5   Summary

The Venn diagram shown in Fig. 1 above summarizes the relation between the three different types of descriptors defined in Secs. 2 - 4 above. We see that they combine to produce 8 different cases. As a simple exercise, the reader may wish to sketch an example of each type of signal.