May 6, 1998 **Name:____________________**

EE 438 **Final Exam** **Spring 1998**

- You have 120 minutes to work the following six problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (30 pts.) Signals and systems
   a. (15) Consider the signal $x[n] = 2^n u[-n]$.
      i. Compute the metrics $x_{\max}$, E_x, x_{rms} for this signal.
      ii. Is it causal, anticausal, or mixed causal?
   b. (15) Consider the system described by the difference equation $y[n] = x[n] - y[n-1]$.
      i. Find the impulse response $h[n]$ for this system.
      ii. Find a simple expression for the frequency response $H(\omega)$ for this system.
      iii. Is the system BIBO stable?

1. (continued)

2. (25 pts.) Sampling and DTFT. A CT signal $x(t)$ with bandwidth 4 kHz is subject to 60 Hz interference from power lines. This signal is digitized at the Nyquist sampling rate to yield a DT signal $x[n]$. You wish to design a simple FIR digital filter of the form $y[n] = ax[n] + bx[n-1] + cx[n-2]$ to block the 60 Hz interference term in the DT signal $x[n]$.

   a. (10) Find the digital frequency of the interfering term.

   b. (15) Find the coefficients a, b, and c for the filter.

2. (continued)

3. (15 pts.) ZT. Consider the signal $x[n] = u[-n] + e^{-n}u[n]$.
   a. (10) Find the ZT for this signal.
   b. (5) Sketch the locations of all poles and zeros and indicate the region of convergence.

3. (continued)

4. (20 pts.) DFT. Consider a signal $x[n]$ of length $N = 3M$, where M is an integer. Now define a new signal

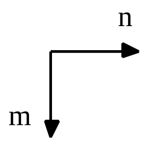
$$y[n] = \begin{cases} x[n], & n/3 \text{ is an integer} \\ 0, & \text{else} \end{cases}$$

Find a simple expression for the N point DFT $Y[k]$ in terms of the N point DFT $X[k]$.

Hint: Use the signal $s_3[n] = \sum_{l=0}^{2} e^{-j2\pi nl/3}$.

4. (continued)

5. (30 pts.) Spatial filtering. Consider the image $x[m,n]$ shown below:



0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1

Suppose we filter this image according to the equation

$$y[m,n] = \tfrac{1}{4}\,x[m,n] + \tfrac{1}{4}\,x[m-1,n] - \tfrac{1}{4}\,x[m,n-1] - \tfrac{1}{4}\,x[m-1,n-1]$$

a. (6) Find the impulse response for this filter.

b. (8) Compute the output image $y[m,n]$.

c. (10) Find a simple expression for the magnitude of the frequency response $|H(\mu,\nu)|$ for this filter.

Hint: This filter is separable.

d. (4) Sketch $|H(\mu,\nu)|$ for the two cases:

i. $\mu = 0$,

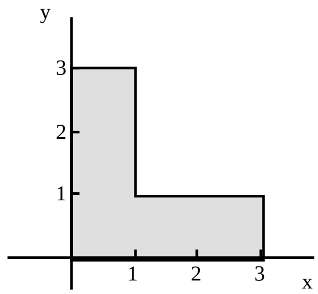
ii. $\nu = 0$,

e. (2) Discuss the relation between your answers to parts b. and d.

5. (continued)

6. (20 pts.) Computed tomography.

a. (6) Consider the object $g(x, y)$ shown below, which has constant attenuation of unity within the shaded area, and attentuation zero elsewhere.



Compute the projection $p_\theta(t)$ for the angles

i. $\theta = 0$,

ii. $\theta = \pi / 2$.

b. (14) An unknown object $g(x, y)$ has Radon transform $p_\theta(t) = e^{-\pi t^2}$. Use the Fourier slice theorem to find $g(x, y)$.

Hint: Note the following Fourier transform pair: $e^{-\pi t^2} \overset{\text{CTFT}}{\leftrightarrow} e^{-\pi f^2}$.

6. (continued)

1.	_______
2.	_______
3.	_______
4.	_______
5.	_______
6.	_______
Total	_______