March 20, 1998 **Name:**_____

EE 438 **Exam No. 2** **Spring 1998**

- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (30 pts.) Consider the causal system defined by the equation $y[n] = x[n] - \frac{1}{2}y[n-1]$, with input $x[n] = (\frac{3}{4})^n u[-n]$.

   a. (12) Find the Z transform $Y(z)$ of the output $y[n]$.

   b. (6) Sketch a plot of the poles and zeros for $Y(z)$, and show its region of convergence.

   c. (12) Using your expression for $Y(z)$, find the response $y[n]$.

1. (continued)

2. (20 pts.) Consider the continuous-time signal $x(t) = \cos(2\pi(1000)t)$.
   a. (5) Find and sketch the CTFT $X(f)$.
   b. (15) Let $x[n] = x(n/10^4)$. Find and sketch the 20 point DFT $X[k]$ of $x[n]$, $0 \le n \le 19$.

   Hint: Use the formulas wherever they are applicable.

2. (continued)

3. (20 pts.) Consider the length N signal $x[n], 0 \le n \le N-1$. Define another length N signal $y[n] = (-1)^n x[n], 0 \le n \le N-1$.

a. (14) Find a simple expression for the N point DFT $Y[k]$ in terms of the N point DFT $X[k]$.

b. (4) Sketch $Y[k]$ for the case where $x[n] = \cos(\pi n / 5)$, and $N = 20$.

c. (2) Comment on the relation between your answer for parts a and b of Problem 2 and your answer to part b of this problem.

3. (continued)

4. (30 pts.) You have a subroutine that computes the radix 2 FFT; but you wish to compute a DFT $X^{(384)}[k]$ of a signal $x[n]$ that is exactly 384 points in length.

   a. (14) Find an expression for $X^{(384)}[k]$ that shows how it can be computed efficiently by using your radix 2 FFT subroutine.

   b. (10) Based on your answer to part a, draw a flow diagram for the algorithm. (Just show your radix 2 FFT as a single block; don't attempt to break it down.)

   c. (6) Determine the number of complex operations required for this computation.

4. (continued)

1. _____

2. _____

3. _____

4. _____

Total _____