Due 11 April 1997 at 5:00 PM

EE 438 — Assignment No. 9 — Spring 1997

1. A Gaussian signal $x(t)$ has mean 0 and variance 10. It is to be quantized with a uniform quantizer and encoded into B bit words.
   - a. Find the range of the quantizer which will result in overload 1% of the time.
   - b. Using the range that you determined above and assuming a uniform distribution for the quantization error, find an expression for the signal-to-noise ratio in dB of the quantized signal as a function of the number of bits B.

2. For each of the following two bivariate density functions $f_{XY}(x,y)$, please do the following:
   - i. Sketch $f_{XY}(x,y)$
   - ii. Compute the moments $\mathrm{E}\{X\}$, $\mathrm{E}\{Y\}$, $\mathrm{E}\{XY\}$, $\mathrm{Var}(X)$, $\mathrm{Var}(Y)$, $\mathrm{Cov}(X,Y)$, and the correlation coefficient ρ_{XY}.
   - iii. Find the marginal densities $f_X(x)$ and $f_Y(y)$.
   - iv. Sketch the marginal densities $f_X(x)$ and $f_Y(y)$.
   - v. Are X and Y independent? Are they uncorrelated?
   - a. $f_{XY}(x,y) = \begin{cases} 4xy, & 0 \le x \le 1, \text{ and } 0 \le y \le 1, \\ 0, & \text{else.} \end{cases}$
   - b. $f_{XY}(x,y) = \begin{cases} 2, & 0 \le x \le 1, \text{ and } 0 \le y \le x, \\ 0, & \text{else.} \end{cases}$

3. A sequence of random variables $X_1, \ldots, X_N$ is identically distributed with mean 0, variance 1, and the following correlation:

$$\mathrm{E}\{X_i X_j\} = \begin{cases} 1, & i = j, \\ \gamma, & |i - j| = 1, \\ 0, & \text{else.} \end{cases}$$

   Here γ is a constant. A new random variable Y is defined by $Y = X_1 + \cdots + X_N$.
   - a. Find the mean of Y.
   - b. Find the variance of Y.

4. Let $X_1, \ldots, X_N$ be an independent, identically distributed sequence of random variables with mean 0 and variance 1. Suppose we form a new sequence of random variables $Y_1, \ldots, Y_M$ according to

$$\begin{aligned} Y_1 &= a_{11}X_1 + \cdots + a_{1N}X_N \\ &\vdots \\ Y_M &= a_{M1}X_1 + \cdots + a_{MN}X_N \end{aligned}$$

   - a. Find the mean, variance, and covariance of the sequence $Y_1, \ldots, Y_M$.

b. By defining vectors $\vec{x} = (X_1, \ldots, X_N)^{\mathrm{T}}$ and $\vec{y} = (Y_1, \ldots, Y_M)^{\mathrm{T}}$ (superscript T denotes transpose), express the relation between $\vec{x}$ and $\vec{y}$ in terms of an appropriately defined matrix $\mathbf{A}$.

c. Define the covariance matrix of the sequence $Y_1, \ldots, Y_M$ as $\Sigma_{\vec{y}\vec{y}} = \mathrm{E}\{\vec{y}\,\vec{y}^T\}$. Find an expression for $\Sigma_{\vec{y}\vec{y}}$ in terms of $\mathbf{A}$.