Due 4 April 1997 at 5:00 PM

EE 438 — Assignment No. 8 — Spring 1997

1. Consider the image $g_a(x,y) = \mathrm{jinc}(x-2.5, y-2.5)$, where the subscript denotes analog.

   a. Calculate and sketch its 2D CSFT $G_a(u,v)$.

   The image is sampled at interval 0.5 in both the x and y directions to yield $g[m,n] = g_a(0.5m, 0.5n)$.

   b. Based on your answer to part a., calculate and sketch the 2D DSFT $G(\mu, \nu)$.

   c. Based on your answer to part b., calculate and sketch the 10×10 2D DFT of $g[m,n]$, $0 \le m \le 9$, $0 \le n \le 9$. You may assume that $g[m,n] = 0$ outside this range.

   Note: Your sketches need only show magnitude.

   Hint: You should only do one actual Fourier transform, and that is for part a.

   d. Write a Matlab routine to calculate the jinc function.

   e. Plot the jinc function along the x axis, and compare with the 2-D sinc function plotted along the x axis.

   f. Generate a mesh plot of the 2D jinc function.

   g. Sample the jinc function as above, and use Matlab to compute the 2D DFT as in part c. Generate a mesh plot of the 2D DFT.

   h. Increase the sampling rate by a factor of 4 in both x and y, and compute the 40×40 2D DFT. Generate a mesh plot of the 2D DFT.

   i. Discuss the significance of your results for this problem.

2. The $M \times M$ point 2D DFT is defined as

$$X[k,l] = \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} x[m,n] e^{-j2\pi(km+\ln)/M}$$

   a. Show that $X[k,l]$ may be computed by performing M 1D length M DFT's of the columns of $x[m,n]$, followed by M 1D length M DFT's of the rows of this intermediate result.

   Assume that the 1D DFT's each require $M \log_2 M$ complex operations (CO's) to compute.

   b. Determine the number of complex operations required to compute the 2D DFT $X[k,l]$. Compare with the number of CO's required for a single M^2 point 1D DFT.

3. Consider a 3×3 FIR filter with coefficients $h[m,n]$

	m		
n	-1	0	1
1	-0.5	0.0	0.5
0	0.0	1.0	0.0
-1	0.5	0.0	-0.5

a. Find a difference equation that can be used to implement this filter.

b. Find the output image that results when this filter is applied to the input image shown below:

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0
0	0	1	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	0	0

c. Find a simple expression for the frequency response $H(\mu, \nu)$ of this filter.

d. Plot $H(\mu, \nu)$ along the μ axis $(\nu = 0)$, along the ν axis $(\mu = 0)$, along the line $\mu = \nu$, and along the line $\mu = -\nu$.

e. Discuss the relation between your answer to part b. and the filter frequency response.