Due 28 February 1997 at 5:00 PM

EE 438 Assignment No. 5 Spring 1997

1. Find expressions for the N point DFT's of the following signals. Simplify your answers as much as possible.

   a. $x[n] = 0.5\,\delta[n-(N-1)/2] - 0.5\,\delta[n-(N+1)/2]$, N odd

   b. $x[n] = \cos(8\pi n / N)$

   c. $x[n] = \cos(3\pi n / N)$

2. In class, we stated the following relation for modulation

$$e^{j2\pi n k_0 / N} x[n] \overset{\text{DFT}}{\leftrightarrow} X[k-k_0].$$

   The objective of this problem is to generalize this result to an arbitrary modulating frequency, not just one that is an integer multiple of $2\pi / N$. Toward this end, define $y[n] = e^{j\omega_0 n} x[n]$, and find a simple expression for the DFT $Y[k]$ in terms of $X[k]$. *Hint:* Substitute the inverse DFT of $X[k]$ for $x[n]$ in the definition of $y[n]$.

3. The signal $x(t) = 10\cos[2\pi(500)t] + 5\cos[2\pi(2500)t]$ is sampled at 4 kHz using an ideal A/D convertor to produce the digital signal $x[n]$. You compute a 512 point DFT $X[k]$ of this signal. Find the approximate values of k and the amplitudes $|X[k]|$ corresponding to the spectral peaks in the analog signal.

4. Let $v[n]$ be a length N compex-valued signal with DFT $V[k]$.

   a. Show that $\text{DFT}\left\{v^*[n]\right\} = V^*[N-k]$.

   Let $x[n]$ and $y[n]$ be two real-valued signals with N point DFT's $X[k]$ and $Y[k]$. Form the complex-valued signal $v[n] = x[n] + jy[n]$.

   b. Find expressions for $x[n]$ and $y[n]$ in terms of $v[n]$.

   c. Combining your answers to Parts a and b, show how the N point DFT's of *two* real signals can be calculated by computing just *one* N point DFT of a complex-valued signal, *i.e.* show how $X[k]$ and $Y[k]$ may be recovered from $V[k]$.

5. Consider the signal

$$x[n] = \cos(\omega_1 n) + a\cos(\omega_2 n) + b\,d[n],$$

   where a and b are constants and $d[n]$ is a sequence of independent Gaussian random variables with zero mean and unit variance.

   a. Write a MATLAB program that will

i. plot x[n],

ii. compute the N point DFT $X[k]$ (using FFT routines available within MATLAB),

iii. plot $|X[k]|$.

Turn in a print-out of your M-file with your homework.

b. Run your program and generate output for the cases shown in the table on the following page. Turn in the plots generated for each case.

c. Discuss the significance of each case.

Case	N	ω_1	a	ω_2	b
1	128	0.44178647	0.0	-	0.0
2	16	0.41724277	0.0	-	0.0
3	128	0.41724277	0.0	-	0.0
4	128	0.41724277	0.1	0.78539816	0.0
5	128	0.41724277	0.1	0.44178647	0.0
6	128	0.41724277	0.1	0.78539816	0.05
7	128	0.41724277	0.1	0.78539816	0.2