Due 31January 1997 at 5:00 PM

EE 438 Assignment No. 2 Spring 1997

1. For each of the following signals, compute the DTFT $X(\omega)$. Simplify your answer as much as possible.

   a. $x[n] = 2^{-|n|}(\mathrm{u}[n+10] - \mathrm{u}[n-10])$,

   b. $x[n] = n2^{-n}\,\mathrm{u}[n]$

   c. $x[n] = \cos(\pi n / 10 + \pi / 5)$.

2. Find an expression for the DTFT $Y(\omega)$ of the output in terms of the DTFT $X(\omega)$ of the input, when y[n] and x[n] are related by (Simplify your answer as much as possible, and assume that x[n] is real-valued.):

   a. $y[n] = j^n x[n]$,

   b. $y[n] = x[n] * x[-n]$,

   c. $y[n] = x[n \bmod 10]$, where $x[n] \neq 0$ only for $0 \le n \le 9$.

   d. $y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

3. Perform the convolution of the following pairs of signals:

   a. $u[n+5] - u[n-5]$ and $2^{-n}u[n]$,

   b. $\delta(2t-1)N$ and $\mathrm{rect}(t)$.

   c $(1-t^2)\mathrm{rect}[(t-1/2)]$ and $\mathrm{rect}[(t+1)/2]$.

4. Use known transform pairs and transform relations to find the CTFT of the following signal, *i.e.* do not directly evaluate any Fourier integrals. Simplify your answer as much as possible.

$$x(t) = \sin(2\pi t)\Lambda(0.5t) \text{ where } \Lambda(t) \stackrel{\Delta}{=} [1-|t|]\mathrm{rect}(t/2)$$

5. Consider a DT LTI system described by the following equation

$$y[n] = x[n] + x[n-1] + x[n-2].$$

   Find the response of this system to the input

$$x[n] = \begin{cases} 1, & n = -1, \\ 1/2, & n = 0, \\ 1/4, & n = -1, \\ 0, & \text{else.} \end{cases}$$

   by the following approaches:

   a. directly substitute $x[n]$ into the difference equation describing the system;

   b. find the impulse response $h[n]$ and convolve it with $x[n]$;

   c. find the frequency response $H(\omega)$ by the following two approaches:

      i. apply the input $e^{j\omega n}$ to the difference equation describing the system,

      ii. find the DTFT of the impulse response,

verify that both methods lead to the same result, then find the DTFT $X(\omega)$ of the input, multiply it by $H(\omega)$ to yield the DTFT $Y(\omega)$ of the output, and finally calculate the inverse DTFT $y[n]$.

Hints:

i. There is no need to simplify the frequency response or the DTFT of the input.

ii. To evaluate the inverse DTFT of $Y(\omega)$, simply put it in the series form $Y(\omega) = \sum_n y[n]\, e^{-j\omega n}$, and identify the terms $y[n]$ in the series.

d. Verify that all three approaches for finding $y[n]$ lead to the same result.

6. Consider the DT LTI system described by the difference equation

$$y[n] = \{x[n] + x[n-1] + x[n-2] + \cdots + x[n-(N-1)]\} / N$$

a. Find expressions for the magnitude $|H(\omega)|$ and phase $\arg[H(\omega)]$ of the frequency response for this system.

b. Use Matlab to plot these quantities for $-\pi \le \omega \le \pi$ for the following values of the constant N:

i. $N = 3$,

ii. $N = 4$,

iii. $N = 10$.

c. Discuss the significance of your results. What happens as N increases?