# Sample Questions for EE 438 Exam No. 2 - Spring 1997

Problem 1.

a. Derive a **complete** flow diagram for a 4 point FFT. Be sure to label all twiddle factors.

b. The signal $x(t) = \cos[2\pi(6000)t]$ is sampled at a rate of 8 kHz, with the first sample at time t = 0.0 sec., and the last sample at time t = 1/8 - 1/8000 sec. to yield the 1000 point DT signal $x[n]$. The DFT $X[k]$ is then computed. Find the indices $k_1$ and $k_2$ where you would observe peaks in the DFT spectrum.

Problem 2.

a Consider the DT LTI system described by the difference equation $y[n] = x[n] + 0.5y[n-1]$. The input to this system is $x[n] = \left(\frac{3}{4}\right)^n u[-n]$. Find the ZT $Y(z)$ of the output. Be sure to specify the region of convergence.

b. Find the inverse ZT $x[n]$ corresponding to $X(z) = 2\dfrac{1-\frac{2}{3}z^{-1}}{1-\frac{4}{3}z^{-1}+\frac{2}{3}z^{-2}}, \quad 1<|z|$

Problem 3.

a. A causal DT system described by the equation $y[n] = x[n] - 0.33y[n-1]$ is subjected to the input $x[n] = \cos(\pi n/4)u[n]$. Find the ZT $Y(z)$ of the output. Be sure to specify the region of convergence for $Y(z)$, and identify the locations of all poles and zeros.

b. A signal $x[n]$ has ZT

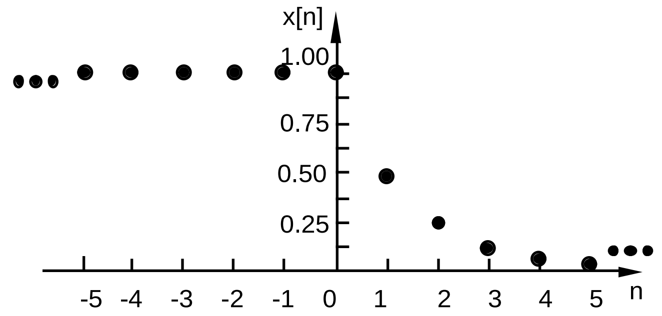
$$X(z) = \frac{1}{(1-0.1z^{-1})(1-0.9z^{-1})}, \quad 0.1<|z|<0.9,$$

Find $x[n]$.

Problem 4.

Z transform (ZT)

a. Find the ZT $X(z)$ for the signal plotted below and sketch the region of convergence.



b. A causal, linear, time-invariant discrete-time system has impulse response $h[n] = 2^{-n}u[n]$. The response to an unknown input $x[n]$ is $y[n] = (-1)^n u[n]$. Find the input $x[n]$.

Problem 5.

Discrete Fourier Transform (DFT)/Fast Fourier Transform Algorithm (FFT)

a. The continuous-time signal $x(t) = \text{sinc}(t)$ is sampled at 101 points evenly spaced between $t = -5.0$ and $t = 5.0$ (including samples at the end-points) to yield the discrete-time signal $x[n]$. Sketch the 101 point DFT $X[k]$ for this signal.

Be sure to show at which frequency sample $k_0$ the cutoff for the spectrum occurs.

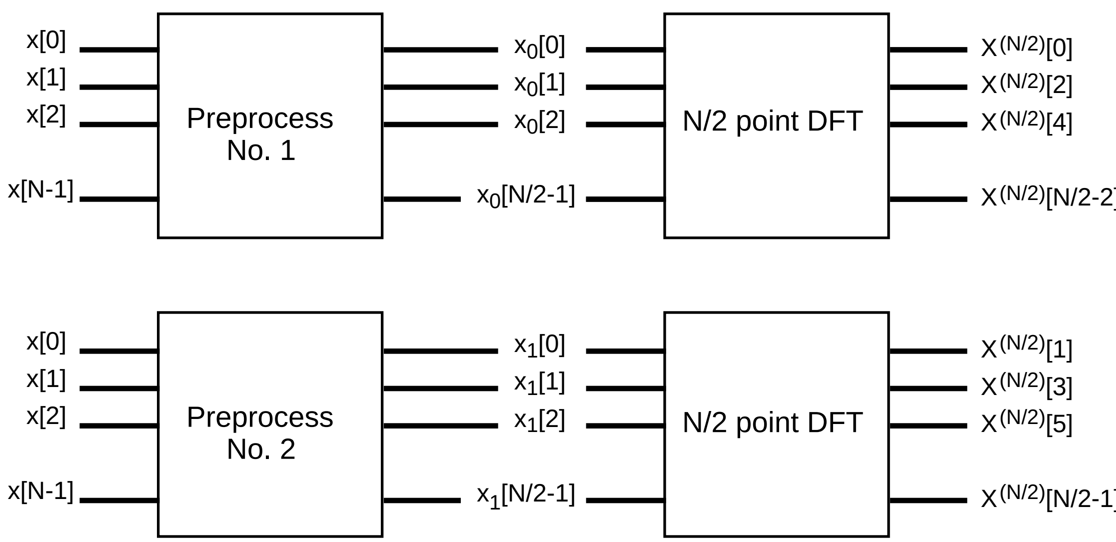
What is the effect of truncating the signal?

**NOTE:** Do not attempt to directly compute the DFT of x[n]. Instead, you should find X(f), the CTFT of x(t), then use relation between CTFT and DTFT and relation between DTFT and DFT to find X[k].

b. The $N$ point DFT is defined as

$$X^{(N)}[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}.$$

Show that if $N$ is even, $X^{(N)}[k]$ can be computed as shown in the following block diagram:




Here the entire $N$ point data sequence $x[n]$ is preprocessed in two different ways to yield two different $N/2$ point data sequences $x_0[n]$ and $x_1[n]$. Taking the $N/2$ point DFT of $x_0[n]$ yields $X^{(N)}[k]$ for even values of k; and taking the $N/2$ point DFT of $x_1[n]$ yields $X^{(N)}[k]$ for odd values of k.

*Hint:* Let $k = 2l$ to get the even values; and let $k = 2l + 1$ to get the odd values, where $l = 0, \ldots, N/2 - 1$.

Be sure to specify precisely what Preprocesses Nos. 1 and 2 are.