April 17, 1997 **Name:____________________**

EE 438 **Exam No. 3** **Spring 1997**

- You have 75 minutes to work the following 5 problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes. However, you may bring with you 3 sheets of formulas handwritten on both sides of one 8.5x11 in. sheet of paper, readable by the unaided eye.
- Calculators are permitted.

1. (20 pts.) Consider the signal $g(x,y) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} \mathrm{sinc}[(x-2k)/0.2,(y-2l)/0.2]$.

   a. Find a *simple* expression for the 2D continuous-space Fourier transform (CSFT) of $g(x,y)$.

   b. Accurately sketch $G(u,v)$.

1. (continued)

2. (20 pts.) Consider a 1-D function $f(x)$ with 1-D CSFT $F(u)$. Let us define a new 2-D function $g(x,y)$ in terms of $f(x)$ according to $g(x,y) = f(x+y)$.

a. For the specific function $f(x) = \text{rect}(x)$, sketch $g(x,y)$.

b. For a general function $f(x)$, find a *simple* expression for the 2-D CSFT $G(u,v)$ of $g(x,y)$ in terms of $F(u)$.

2. (continued)

3. (20 pts.) Consider the digital image $f[m,n]$ shown below.

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1

a. Find the result of filtering this image with the filter $h[k,l]$ given by

		k = –1	k = 0	k = 1
	–1	–1 / 8	1 / 2	–1 / 8
l	0	–1 / 4	1	–1 / 4
	1	–1 / 8	1 / 2	–1 / 8

b. Find a *simple* expression for the frequency response (2D-DSFT) $H(\mu, \nu)$ for this filter. Plot $H(\mu, \nu)$ along the μ and ν axes.

c. Discuss the spatial domain properties of this filter, and relate these properties to its frequency response.

3. (continued)

4. (20 pts.) If the camera moves while the shutter is open, the image will be blurred. For horizontal motion with constant velocity, such blur can be modeled according to

$$g[m,n] = \sum_{k=0}^{N-1} f[m,n-k], \qquad (1)$$

where $f[m,n]$ denotes the desired unblurred image, and $g[m,n]$ denotes the blurred image that is recorded on film (and subsequently digitized). The integer constant N depends on the velocity of the camera and the length of the exposure.

The objective of this problem is to devise a filter for deblurring $g[m,n]$, and thus recovering $f[m,n]$. To do this, we need the 2-D Z transform (ZT) defined as

$$G(z_1,z_2) = \sum_m \sum_n g[m,n] z_1^{-m} z_2^{-n} .$$

a. Use the 2D ZT to derive an expression for the transfer function $H(z_1,z_2)$ corresponding to the blur defined by Eq. (1).

b. From your answer to part a, find a difference equation that will yield as its output $f[m,n]$, when the input is $g[m,n]$.

4. (continued)

5. (20 pts.) Consider 2 random variables X and Y which both have mean 1 and variance 2. Their correlation coefficient is 0.5. We define two new random variables U and V according to

$$U = X + Y$$
$$V = X - Y$$

a. Find their means μ_U and μ_V.

b. Find their variances σ_U^2 and σ_V^2 .

c. Find their correlation coefficient ρ_{UV} .

5. (continued)

1 .	**_______**
2 .	**_______**
3 .	**_______**
4 .	**_______**
5 .	**_______**
Total	**_______**