March 20, 1997 **Name:** Solution

EE 438 **Exam No. 2** **Spring 1997**

- You have 75 minutes to work the following five problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes. However, you may bring with you 2 sheets of formulas handwritten on both sides of one 8.5x11 in. sheet of paper, readable by the unaided eye.
- Calculators are permitted.

1. (20 pts.) A signal $x[n]$ has Z transform $X(z) = \dfrac{2}{(1-0.5z^{-1})(1+z^{-1})}$, $0.5 < |z| < 1$.

   Find $x[n]$.

$$X(z) = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+z^{-1}} \qquad \frac{1}{2} < |z| < 1$$

$$A = \frac{4}{3} \qquad B = \frac{2}{3}$$

$$X(z) = \frac{(4/3)}{1-\frac{1}{2}z^{-1}} + \frac{(2/3)}{1+z^{-1}} \qquad \frac{1}{2} < |z| < 1$$

$$x(n) = \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)^n u(n) - \left(\frac{4}{3}\right)(-1)^n u(-n-1)$$

2. (20 pts.) The output $y[n] = \frac{1}{2}\left[(\frac{1}{2})^n + (-\frac{1}{2})^n\right]u[n]$ is observed from a LTI DT system described by the difference equation $y[n] = x[n] - \frac{1}{2}y[n-1]$. Find a simple expression for the input $x[n]$ that generated this output.

$$y(n) = x(n) - \frac{1}{2}y(n-1)$$

$$Y(z) = X(z) - \frac{1}{2}z^{-1}Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$y(n) = \frac{1}{2}\left(\frac{1}{2}\right)^n u(n) + \frac{1}{2}\left(-\frac{1}{2}\right)^n u(n)$$

$$Y(z) = \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) + \frac{1}{2}\left(\frac{1}{1+\frac{1}{2}z^{-1}}\right)$$

$$|z| > \frac{1}{2}$$

$$X(z) = \frac{Y(z)}{H(z)}$$

$$= \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) + \frac{1}{4}\left(\frac{z^{-1}}{1-\frac{1}{2}z^{-1}}\right) + \frac{1}{2}$$

$$|z| > \frac{1}{2}$$

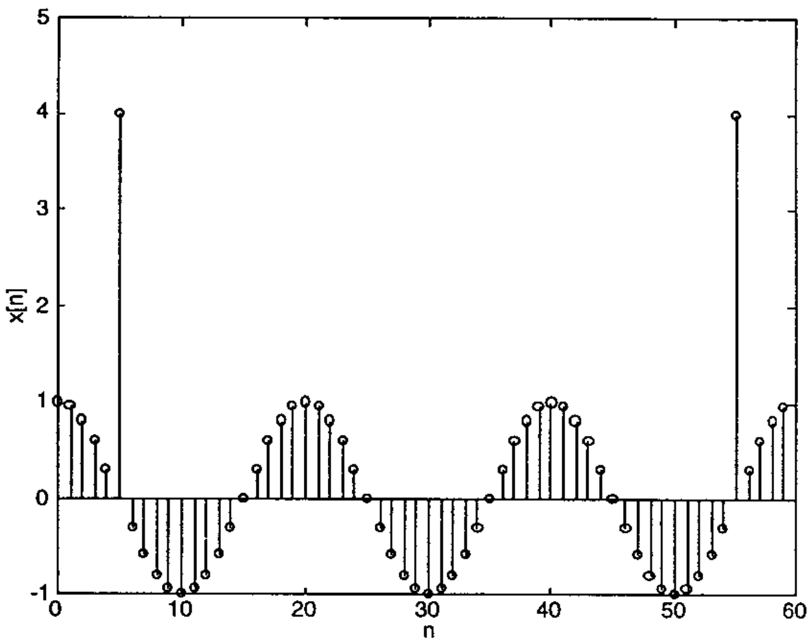
$$= \ldots = \frac{1}{1-\frac{1}{2}z^{-1}} \qquad |z| > \frac{1}{2}$$

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

3. (20 pts.) Consider the signal $x[n]$ shown below.

   a. Find a simple closed form expression for the 60 point DFT $X[k]$ of this signal.

   b. Sketch $X[k]$.



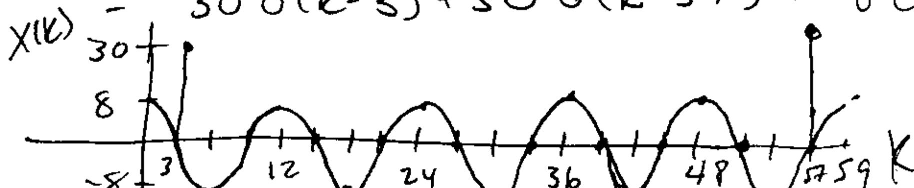
$$x(n) = \cos\left(\frac{2\pi n}{20}\right) + 4\delta(n-5) + 4\delta(n-55)$$

$$X(k) = \frac{60}{2}\left(\delta(k-3) + \delta(k-57)\right) + 4e^{-j\frac{2\pi k5}{60}} + 4e^{-j2\pi k\,55/60}$$

$$= 30\delta(k-3) + 30\delta(k-57) + 4\left(e^{-j\frac{2\pi k}{12}} + e^{-j\frac{2\pi k 11}{12}}\right)$$

$$= 30\delta(k-3) + 30\delta(k-57) + 4\left(e^{-j\frac{2\pi k}{12}} + e^{+j\frac{2\pi k}{12}}\right)$$

$$= 30\delta(k-3) + 30\delta(k-57) + 8\cos\left(\frac{2\pi k}{12}\right)$$



4. (20 pts.) Derive the complete flow diagram for a 6 point FFT. Be sure to specify all twiddle factors.

$$X^{(6)}(k) = \sum_{n=0}^{5} x(n)\, e^{-j2\pi kn/6}$$

$$= \sum_{n=0}^{2} x(2n)\, e^{-j2\pi k2n/6} + \sum_{n=0}^{2} x(2n+1)\, e^{-\frac{j2\pi k(n2+1)}{6}}$$

$$= \underbrace{\sum_{n=0}^{2} x(2n)\, e^{-\frac{j2\pi kn}{3}}}_{\text{3pt FFT} = X_0^{(3)}(k)} + e^{-j\frac{2\pi k}{6}} \underbrace{\sum_{n=0}^{2} x(2n+1)\, e^{-j\frac{2\pi kr}{3}}}_{\text{3pt FFT} = X_1^{(3)}(}$$

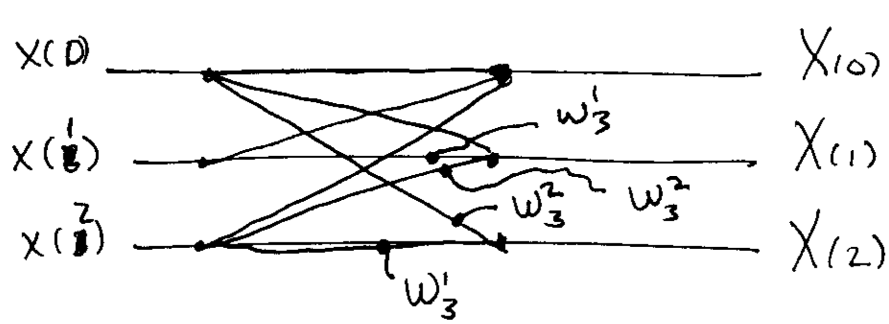
input = x(0), x(2), x(4)

input = x(1), x(3), x(

$$= X_0^{(3)}(k) + W_6^k\, X_1^{(3)}(k)$$

3pt FFT – $X^{(3)}(k) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi kn/3}$

$$X^{(3)}(k) = x(0)\, e^{-j2\pi k0/3} + x(1)\, e^{-j2\pi k/3} + x(2)\, e^{-\frac{j2\pi k2}{3}}$$



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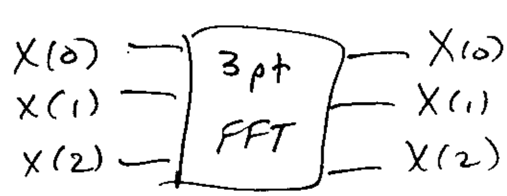
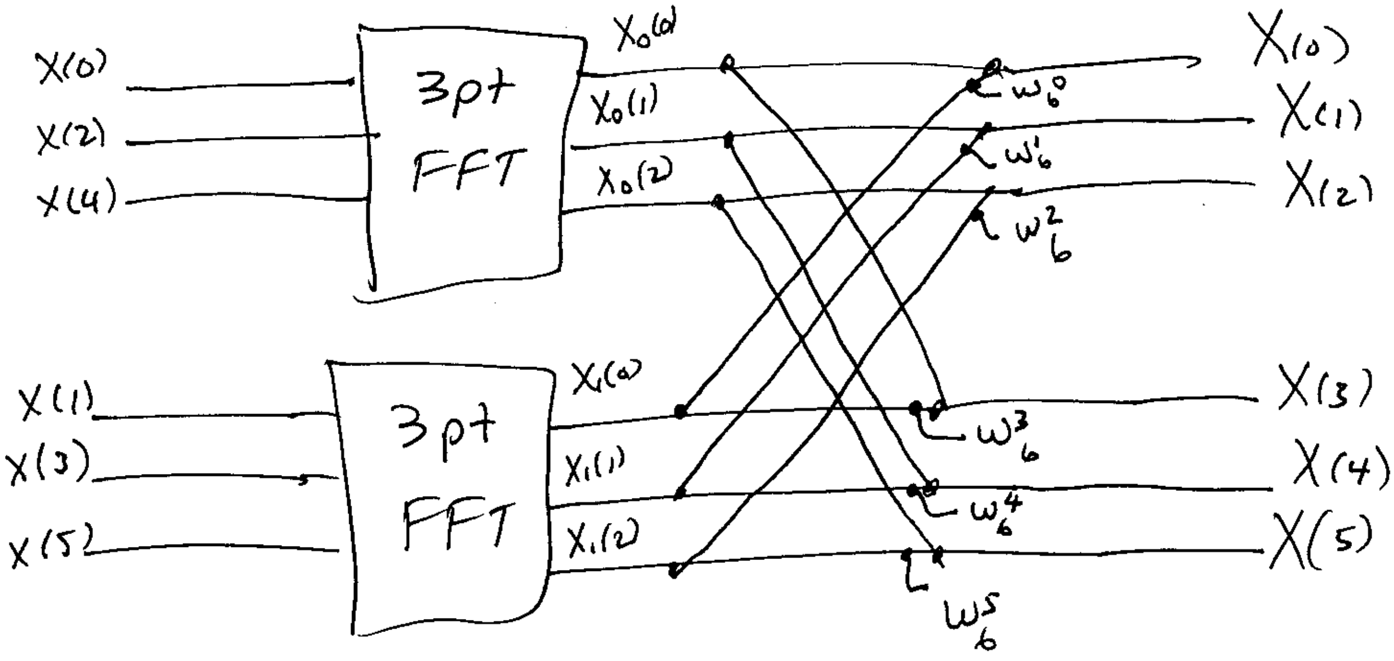


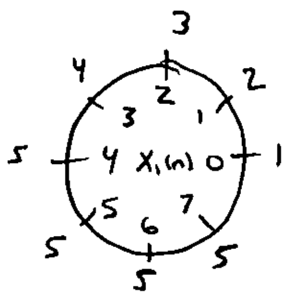
4. (continued)

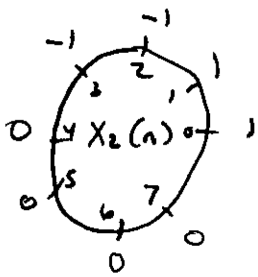

x(0)
x(2)
x(4)
3pt FFT
$X_0(0)$
$X_0(1)$
$X_0(2)$
x(1)
x(3)
x(5)
3pt FFT
$X_1(0)$
$X_1(1)$
$X_1(2)$
W_6^0
W_6^1
W_6^2
W_6^3
W_6^4
W_6^5
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)

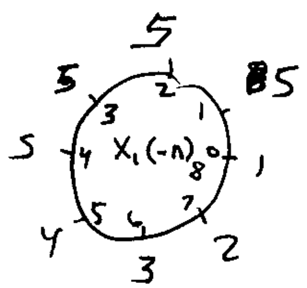
5. (20 pts.) Compute the circular convolution with period 8 of the following two signals:

n	0	1	2	3	4	5	6	7
$x_1[n]$	1	2	3	4	5	5	5	5
$x_2[n]$	1	1	-1	-1	0	0	0	0

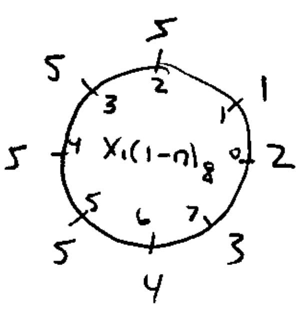
$$x_1(n) \overset{8pt}{\circledast} x_2(n) = y_8(n)$$







$$y_8(0) = x_1(-n)_8 \cdot x_2(n)$$
$$= (1)+(5)+(-5)+(-5) = -4$$



$$y_8(1) = 2 + 1 - 5 - 5 = -7$$

⋮ ⋮

n	0	1	2	3	4	5	6	7
$y_8(n)$	-4	-7	-1	4	4	3	1	0