16 February 2011 **Name:**__________________

ECE 438 **Exam No. 1** **Spring 2011**

- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are **not** permitted.

1. (25 pts.) Consider the linear, time-invariant system defined by the difference equation

$$y[n] = \frac{1}{2}\left(x[n] - x[n-2]\right).$$

a. (5) For each of the following three input signals $x_i[n],\ i = 1,2,3$, find the corresponding response $y_i[n]$ of the system.

n	$\ldots$	-1	0	1	2	3	4	$\ldots$
$x_1[n]$	$\ldots$	1	1	1	1	1	1	$\ldots$
$x_2[n]$	$\ldots$	$-j$	1	j	-1	$-j$	1	$\ldots$
$x_3[n]$	$\ldots$	-1	1	-1	1	-1	1	$\ldots$

b. (5) Find a simple expression for the frequency response $H(\omega)$ of this system.

c. (5) Based on your answer to part (b), find simple expressions for the magnitude $|H(\omega)|$ and phase $\underline{/H(\omega)}$ of the frequency response of this system.

d. (5) Based on your answer to part (c), carefully sketch the magnitude $|H(\omega)|$ and phase $\underline{/H(\omega)}$ of the frequency response of this system.

e. (5) Each of the three input signals $x_i[n],\ i = 1,2,3$ in part (a) can be expressed in the form of a complex exponential signal, i.e. $x_i[n] = e^{j\omega_i n}$ for a particular digital frequency ω_i. For each signal $x_i[n]$, find the corresponding frequency ω_i and from your plots in part (d) evaluated at frequency ω_i and the relationship $y_i[n] = H(\omega_i)e^{j\omega_i n}$ show that the output signal $y_i[n]$ matches that which you obtained in part (a).

(a)

n	$\ldots$	-1	0	1	2	3	4	$\ldots$
$x_1[n]$	$\ldots$	1	1	1	1	1	1	$\ldots$
$y_1[n]$	$\ldots$	0	0	0	0	0	0	$\ldots$
$x_2[n]$	$\ldots$	$-j$	1	j	-1	$-j$	1	$\ldots$
$y_2[n]$	$\ldots$	$-j$	1	j	-1	$-j$	1	$\ldots$
$x_3[n]$	$\ldots$	-1	1	-1	1	-1	1	$\ldots$
$y_3[n]$	$\ldots$	0	0	0	0	0	0	$\ldots$

1. (continued)

(b) $h[n] = \frac{1}{2}\{\delta[n] - \delta[n-2]\} \Rightarrow H(\omega) = \frac{1}{2}\{1 - e^{-2j\omega}\}$

$$= \frac{1}{2}e^{-j\omega}\{e^{j\omega} - e^{-j\omega}\} \times \frac{2j}{2j}$$

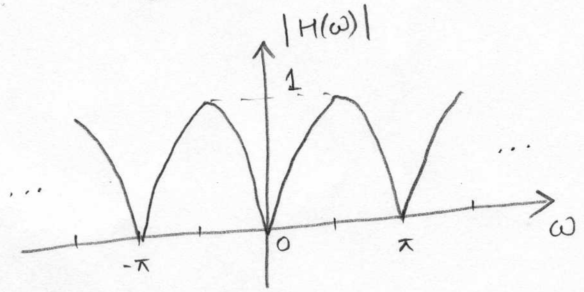
$$= je^{-j\omega}\sin\omega \quad , \quad -\pi < \omega \le \pi$$

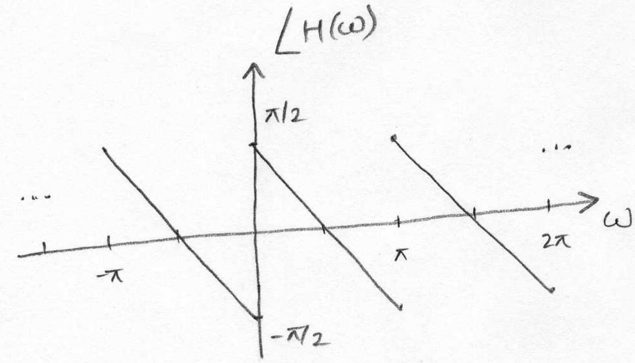
(c) $|H(\omega)| = |\sin\omega|$

$$\angle H(\omega) = \angle j + \angle e^{-j\omega} + \angle \sin\omega$$

$$= \begin{cases} \frac{\pi}{2} - \omega + 0 & , \text{ if } \sin\omega \ge 0 \\ \frac{\pi}{2} - \omega \pm \pi & , \text{ if } \sin\omega < 0 \end{cases}$$

(d)





(e) $x_1[n] = e^{j0n} \quad , \omega_1 = 0 \qquad |H(\omega)|_{\omega=0} = 0$

$$\therefore y_1[n] = H(\omega)|_{\omega=0}\, e^{j0n} = 0$$

$x_2[n] = e^{j\frac{\pi}{2}n} \quad , \omega_2 = \frac{\pi}{2} \qquad |H(\omega)|_{\omega=\pi/2} = 1 \ , \ \angle H(\omega)|_{\omega=\pi/2} = 0$

$$\therefore y_2[n] = H(\omega)|_{\omega=\pi/2}\, e^{j\pi/2\, n} = e^{j\frac{\pi}{2}n} = \{\ldots, \underset{\substack{\uparrow \\ n=0}}{1}, j, -1, -j, 1, \ldots\}$$

$x_3[n] = e^{j\pi n} \quad , \omega_3 = \pi \qquad |H(\omega)|_{\omega=\pi} = 0$

$$\therefore y_3[n] = H(\omega)|_{\omega=\pi}\, e^{j\pi n} = 0$$

$y_i[n]$ from (e) match those from (a)

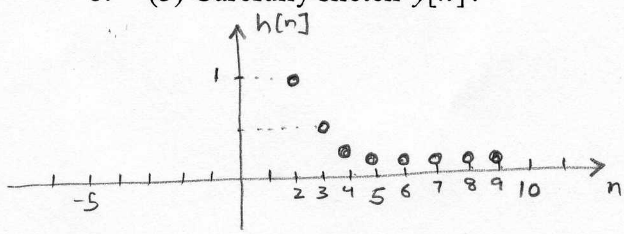
2. (25 pts.) Consider a linear, shift-invariant system with impulse response

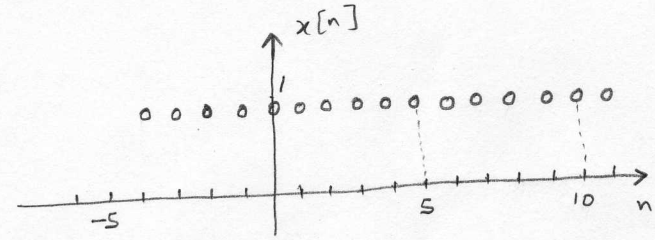
$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^{n-2}, & 2 \le n \le 9 \\ 0, & \text{else} \end{cases} \quad \text{and input} \quad x[n] = \begin{cases} 1, & -4 \le n \le 11 \\ 0, & \text{else} \end{cases}$$

a. (22) Find an expression for the output $y[n]$ by evaluating the convolution sum $y[n] = \sum_{k=-\infty}^{\infty} h[n-k]x[k]$. Simplify your answer as much as possible.

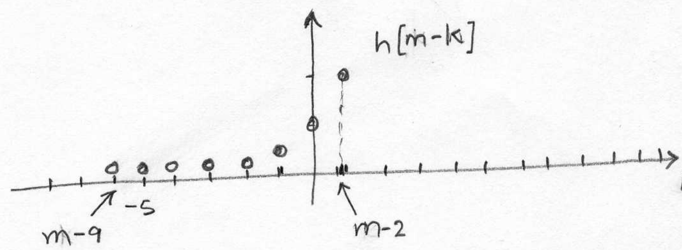
b. (3) Carefully sketch $y[n]$.

(a.)





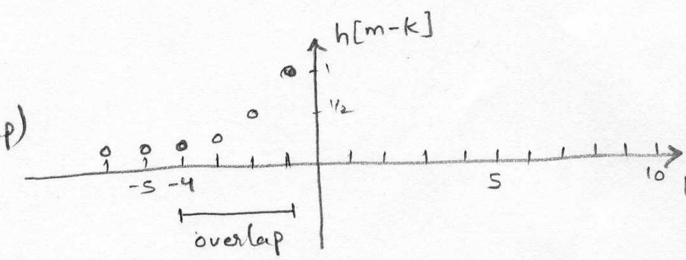
$$y[m] = \sum_{k=-\infty}^{\infty} h[m-k]\,x[k]$$



Case 1: $m < -2$

$m-2 < -4 \Rightarrow$ no overlap

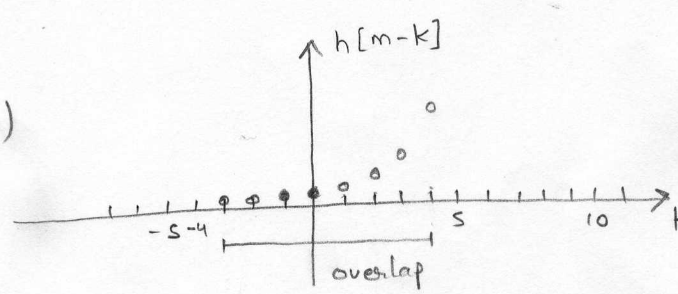
$y[m] = 0 \quad , \; m < -2$



Case 2: $-2 \le m < 5$ (partial overlap)

$$y[m] = \sum_{k=-4}^{m-2} \left(\frac{1}{2}\right)^{(m-k-2)}$$

$$= 2\left\{1 - \left(\frac{1}{2}\right)^{m+3}\right\} \quad , \; -2 \le m < 5$$



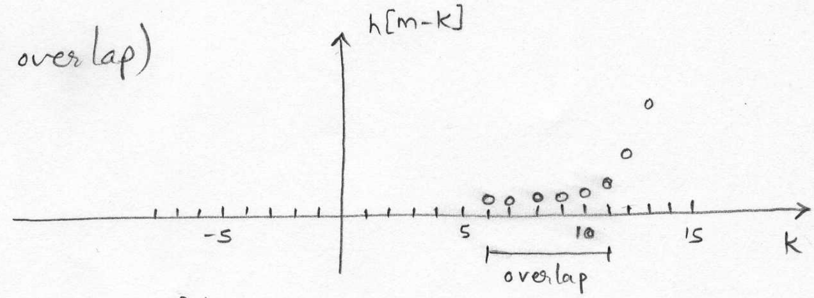
Case 3: $5 \le m < 14$ (full overlap)

$$y[m] = 2\left\{1 - \left(\frac{1}{2}\right)^{8}\right\} \quad , \; 5 \le m < 14$$

$$= \sum_{k=m-9}^{m-2} \left(\frac{1}{2}\right)^{m-k-2}$$

2. (continued)

Case 4: $14 \le m < 21$ (partial overlap)



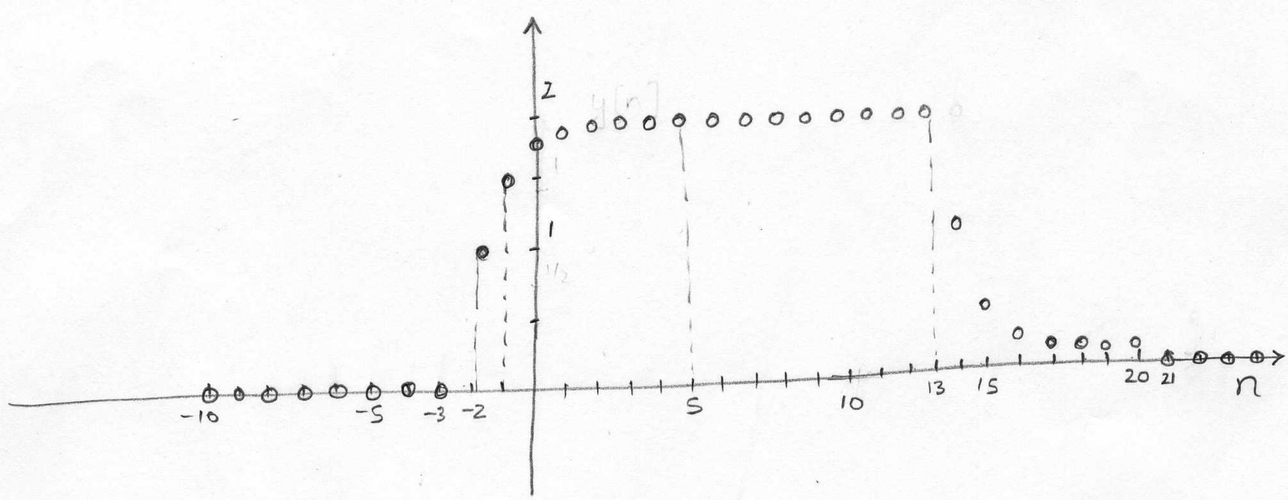
$$y[m] = \sum_{k=m-9}^{11} \left(\frac{1}{2}\right)^{(m-k-2)}$$

$$= \left(\frac{1}{2}\right)^{m-14} \left\{1-\left(\frac{1}{2}\right)^{20-m+1}\right\}, \quad 14 \le m < 21$$

Case 5: $21 \le m$ (no overlap)

$$y[m] = 0, \quad m \ge 21$$

(b)



3. (25) Consider the signal $x(t) = \cos(\pi t)\,\mathrm{rect}(t/2)$.

   a. (5) Carefully and accurately sketch $x(t)$.

   b. (4) Find an expression for the continuous-time Fourier transform (CTFT) $X(f)$ of $x(t)$. Your answer should be simplified as much as possible, and should not contain any operators.

   c. (4) Carefully and accurately sketch $X(f)$.

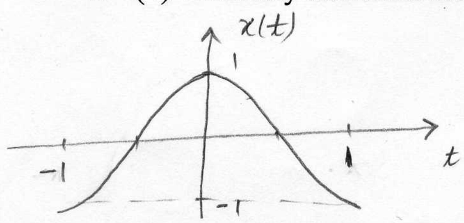
Let $y(t) = \sum_{k=-\infty}^{\infty} x(t-2k)$

   d. (4) Carefully and accurately sketch $y(t)$.

   e. (4) Find an expression for the continuous-time Fourier transform (CTFT) $Y(f)$ of $y(t)$. Your answer should be simplified as much as possible, and should not contain any operators.

   f. (4) Carefully and accurately sketch $Y(f)$.

(a)

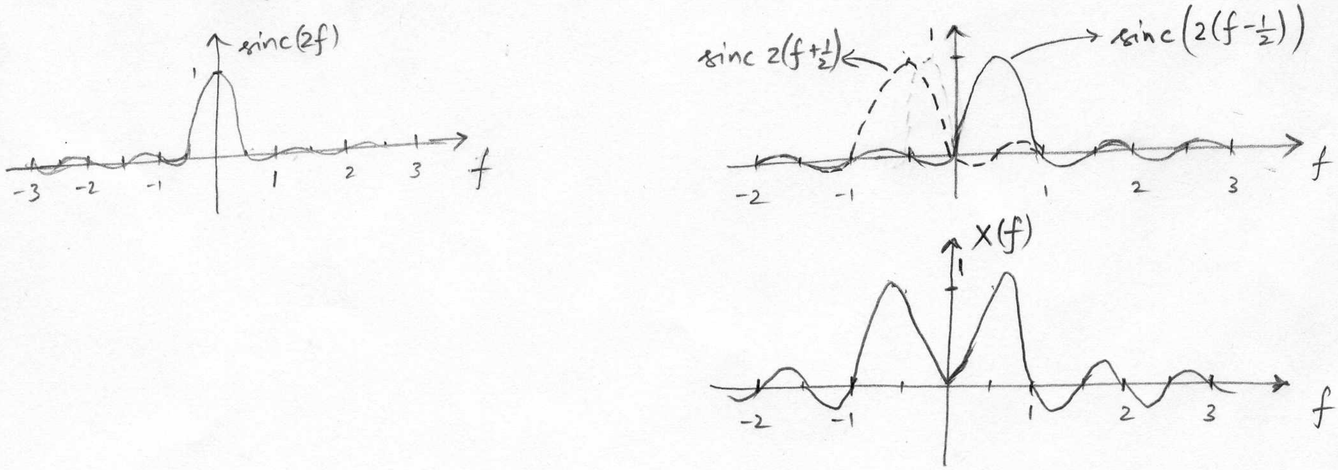


(b) $X(f) = \mathcal{F}\{\cos(\pi t)\} * \mathcal{F}\{\mathrm{rect}(t/2)\}$

$= \frac{1}{2}\{\delta(f-\tfrac{1}{2}) + \delta(f+\tfrac{1}{2})\} * 2\,\mathrm{sinc}(2f)$

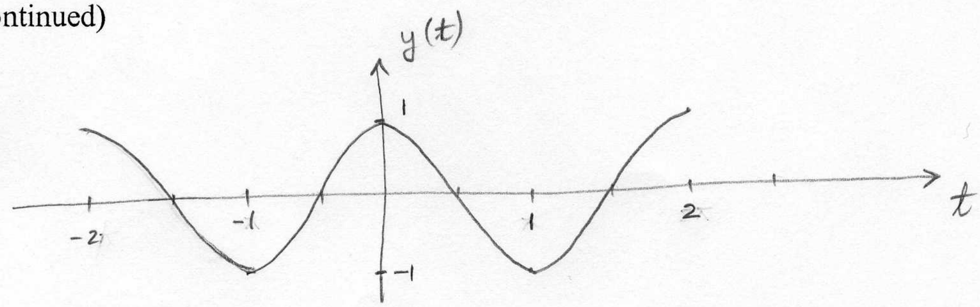
$= \mathrm{sinc}(2(f-\tfrac{1}{2})) + \mathrm{sinc}(2(f+\tfrac{1}{2}))$

(c)



3. (continued)

(d)



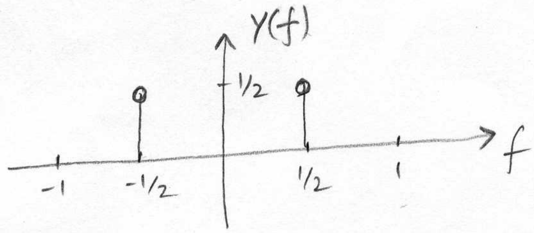
(e) $y(t) = rep_2[x(t)]$

$\therefore Y(f) = \frac{1}{2} comb_{1/2}[X(f)]$

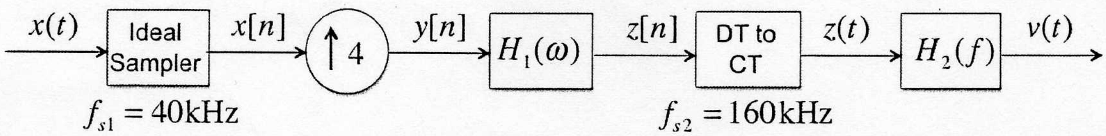
$= \frac{1}{2} comb_{1/2}[sinc\, 2(f-\frac{1}{2}) + sinc\, 2(f+\frac{1}{2})]$

$= \frac{1}{2}\{\delta(f-\frac{1}{2}) + \delta(f+\frac{1}{2})\}$

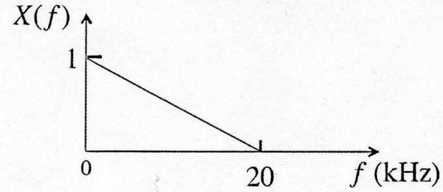
(f)



4. (25 pts) Consider the system shown below:



The CTFT $X(f)$ of the signal $x(t)$ is given by

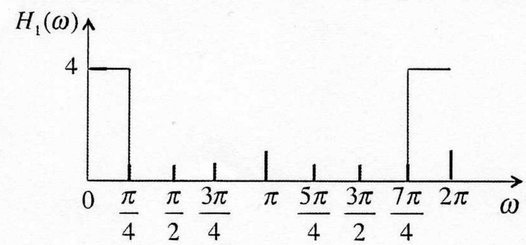


The "Ideal Sampler" generates the DT signal $x[n] = x(nT_1)$, where $T_1 = f_{s1}^{-1} = 1/(40 \times 10^3)$.

a) (5) Carefully sketch the DTFT $X(\omega)$ of $x[n]$.

b) (5) Carefully sketch $Y(\omega)$.

The frequency response $H_1(\omega)$ of the digital filter is given by



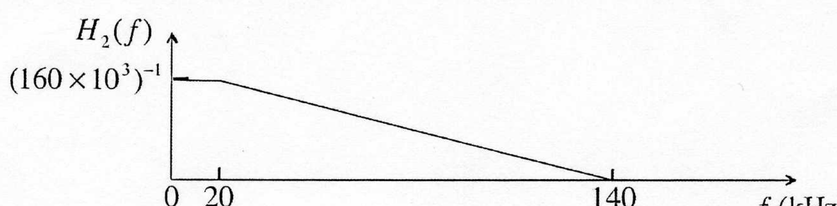
c) (5) Carefully sketch the DTFT $Z(\omega)$ of the filter output $z[n]$.

The operation of the module "DT to CT" is defined as $z(t) = \sum_{n=-\infty}^{\infty} z[n]\delta(t - nT_2)$,

where $T_2 = f_{s2}^{-1} = 1/(160 \times 10^3)$.

d) (5) Carefully sketch the CTFT $Z(f)$ of the signal $z(t)$.

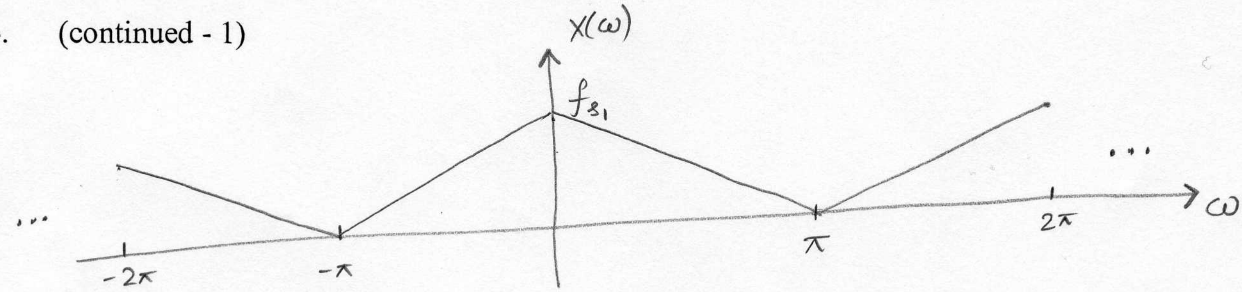
The frequency response $H_2(f)$ of the final analog filter is given by



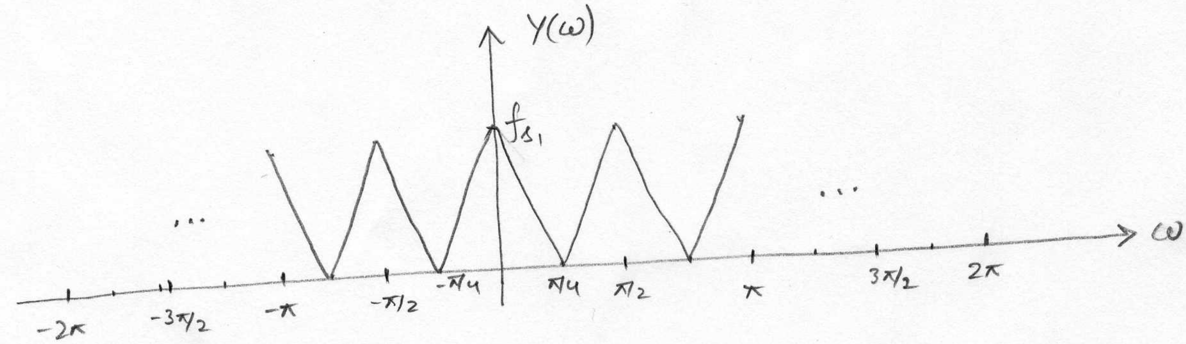
e) (5) Carefully sketch the CTFT $V(f)$ of the final output $v(t)$ of this system.

4. (continued - 1)

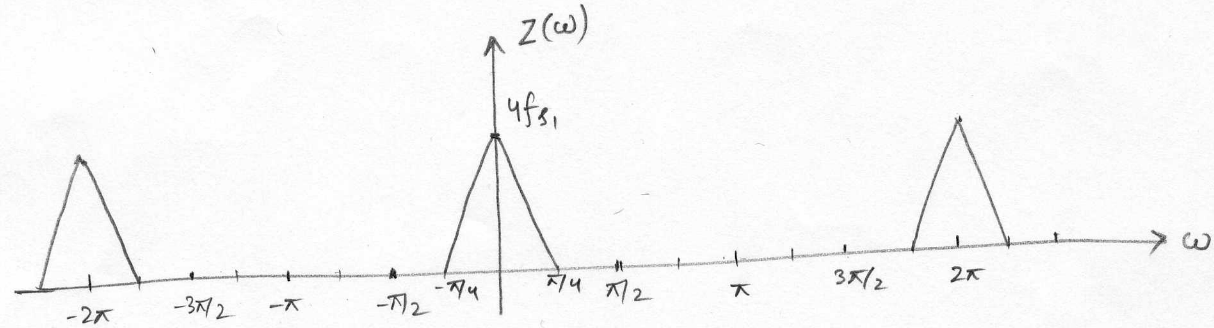
(a)



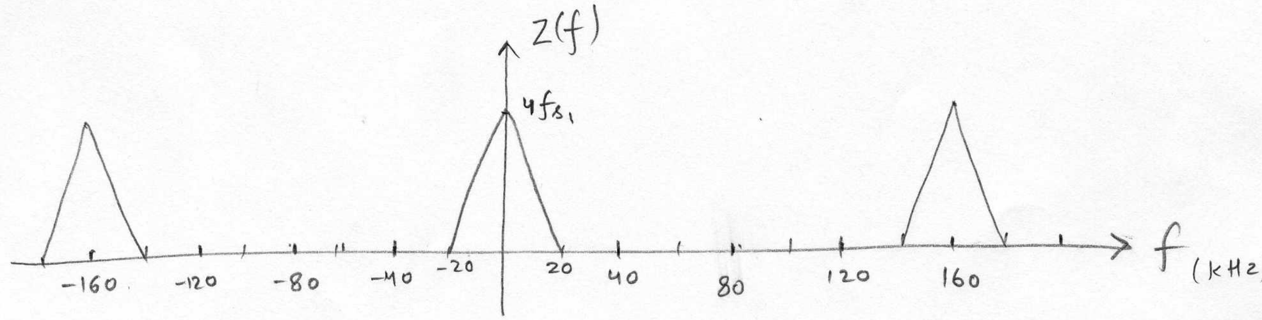
(b)



(c)



(d)



(e)