16 April 2008 **Name:** Key

ECE 438 **Exam No. 3.** **Spring 2008**

- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (20 pts.) Consider a wide-sense stationary random process $X[n]$ with mean $\mu_X \equiv 1$ and autocorrelation function

$$r_{XX}[n] = \begin{cases} 2, & |n| \le 1 \\ 1, & \text{else} \end{cases}.$$

Suppose we filter $X[n]$ to generate a new random process $Y[n]$, according to the equation $Y[n] = \frac{1}{2}(X[n] - X[n-1])$.

a. (5) Find the mean $\mu_Y[n] = E\{Y[n]\}$ of the output process

b. (13) Find the autocorrelation $r_{YY}[m,n] = E\{Y[m]Y[n]\}$ of the output process.

c. (2) Is the process $Y[n]$ wide-sense stationary?

(a) $\mu_y[n] = E[Y[n]] = E[\frac{1}{2}(X[n] - X[n-1])]$

$= \frac{1}{2}\{E[X[n]] - E[X[n-1]]\}$

$= \frac{1}{2}\{1 - 1\}$ since $X[n]$ is wide-sense stationary

$= 0$

(b) $r_{yy}[m,n] = E\{Y[m]Y[n]\}$

$= E\{\frac{1}{2}(X[m] - X[m-1])\frac{1}{2}(X[n] - X[n-1])\}$

$= \frac{1}{4}E\{X[m]X[n] - X[m]X[n-1] - X[m-1]X[n] + X[m-1]X[n-1]\}$

using fact that $X[n]$ is wide-sense stationary

$= \frac{1}{4}\{r_{xx}[m-n] - r_{xx}[m-n+1] - r_{xx}[m-1-n] + r_{xx}[m-\cancel{1}-n+\cancel{1}]\}$

$= \frac{1}{4}\{r_{xx}[m-n] - r_{xx}[m-n+1] - r_{xx}[m-1-n] + r_{xx}[m-n]\}$

$= \frac{1}{4}\{2r_{xx}[m-n] - r_{xx}[m-n+1] - r_{xx}[m-n-1]\}$

$\Rightarrow r_{yy}[m,n] = r_{yy}[m-n]$ let $k = m-n$

$= r_{yy}[k] = \frac{1}{4}\{2r_{xx}[k] - r_{xx}[k+1] - r_{xx}[k-1]\}$

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1. (continued)

(b) $r_{yy}[k] = \frac{1}{4}\{2r_{xx}[k] - r_{xx}[k+1] - r_{xx}[k-1]\}$

for $k > 2$:

$r_{yy}[k] = \frac{1}{4}\{2\cdot 1 - 1 - 1\} = 0$

for $k = 2$:

$r_{yy}[2] = \frac{1}{4}\{2\cdot 1 - 1 - 2\} = -\frac{1}{4}$

for $k = 1$:

$r_{yy}[1] = \frac{1}{4}\{2\cdot 2 - 1 - 2\} = \frac{1}{4}$

for $k = 0$:

$r_{yy}[0] = \frac{1}{4}\{2\cdot 2 - 2 - 2\} = 0$

by symmetry:

for $k < -2$: $r_{yy}[k] = 0$

for $k = -2$: $r_{yy}[k] = -\frac{1}{4}$

for $k = -1$: $r_{yy}[k] = \frac{1}{4}$

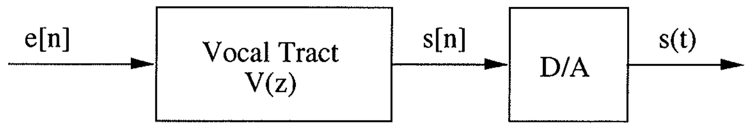
$$\therefore \quad r_{yy}[m,n] = r_{yy}[m-n] = \begin{cases} 0 & |m-n| > 2 \\ -\frac{1}{4} & |m-n| = 2 \\ 0 & |m-n| = 0 \\ \frac{1}{4} & |m-n| = 1 \end{cases}$$

(c) since $E[Y[n]] = 0 \ \forall n$ (ie. mean is a constant $\forall n$)

and $r_{yy}[m,n] = r_{yy}[m-n]$

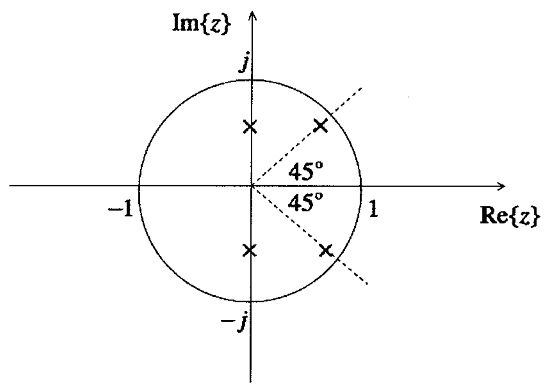
$\Rightarrow Y[n]$ is wide-sense stationary

2. (25 pts.) A digital synthesizer for voiced speech shown below operates at an 8 kHz sampling rate.



The excitation is given by $e[n] = \sum_{k=-\infty}^{\infty} \delta[n-100k]$

The vocal tract transfer function $V(z)$ has poles and zeros at the locations shown below:



a. (7) What is the pitch period in seconds?

b. (8) Find the formant frequencies in Hz, and rank them according to their strength, *i.e.* how peaked the vocal tract response is at the corresponding frequency.

c. (5) Sketch approximately what a *wideband* spectrogram would look like for this utterance. Be sure to label the pitch and formant information appropriately.

d. (5) Sketch approximately what a *narrowband* spectrogram would look like for this utterance. Be sure to label the pitch and formant information appropriately.

Note: Your answers to parts b)-d) above should be based on a graphical interpretation of the pole-zero plot above. You do *not* need to find an expression for $V(z)$.

(a) $N_p = 100$ samples per pitch period

$$T_p = \frac{N_p}{f_s} = \frac{100}{8000} = 12.5 \text{ ms}$$

2. (continued)

(b) $\angle p_1 = 45°$

$F_1 = \frac{45°}{360°} \cdot f_s = \frac{1}{8} 8000 = 1000 \text{ Hz}$

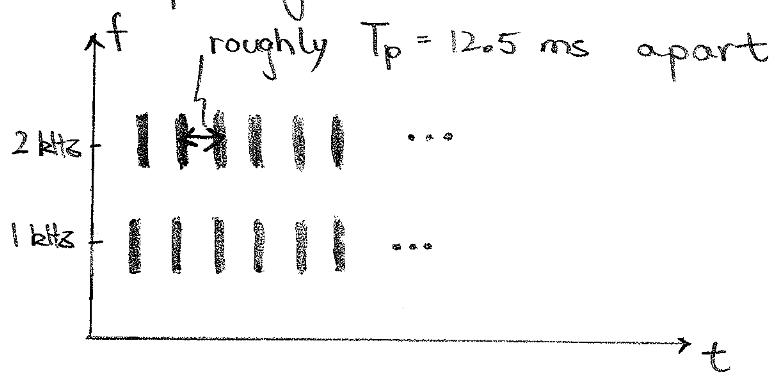
$\angle p_2 = 90°$

$F_2 = \frac{90°}{360°} \cdot f_s = \frac{1}{4} 8000 = 2000 \text{ Hz}$

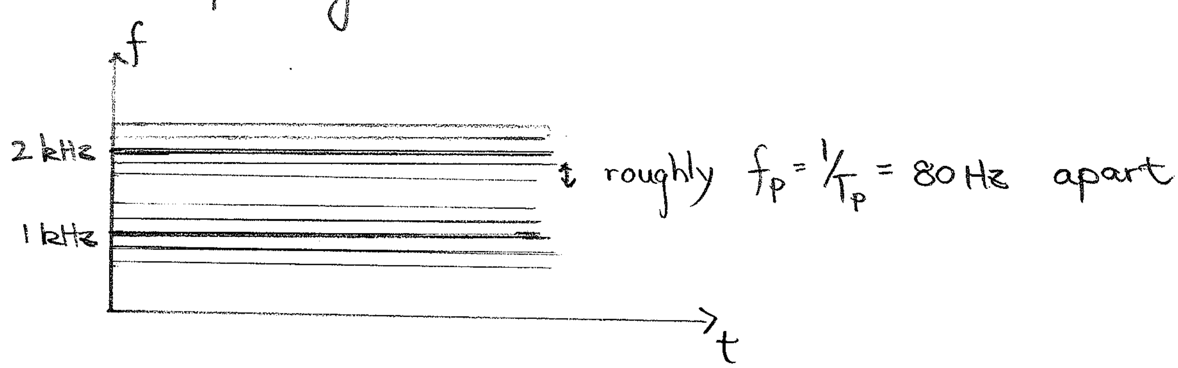
p_1 is closer to unit circle than p_2

$\therefore$ formant at 1 kHz is stronger than formant at 2 kHz

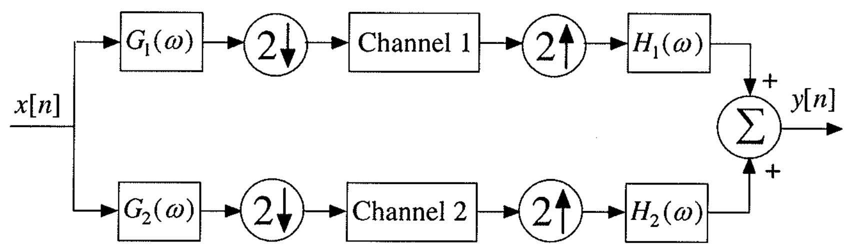
(c) wideband spectrogram



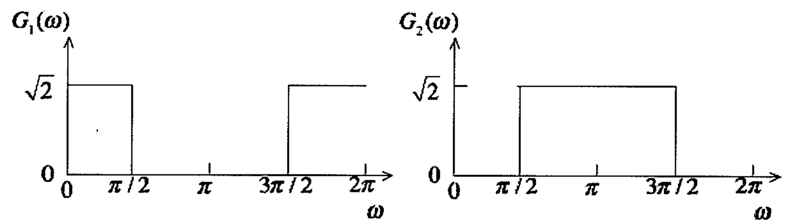
(d) narrowband spectrogram



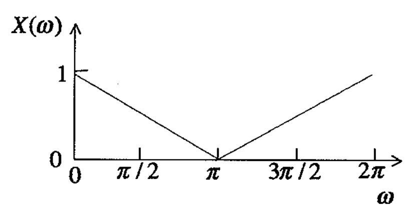
3. (30 pts.) Consider the two-channel filter bank shown below



where $H_1(\omega) = G_1(\omega)$ and $H_2(\omega) = G_2(\omega)$; and the frequency responses of these filters are given by:

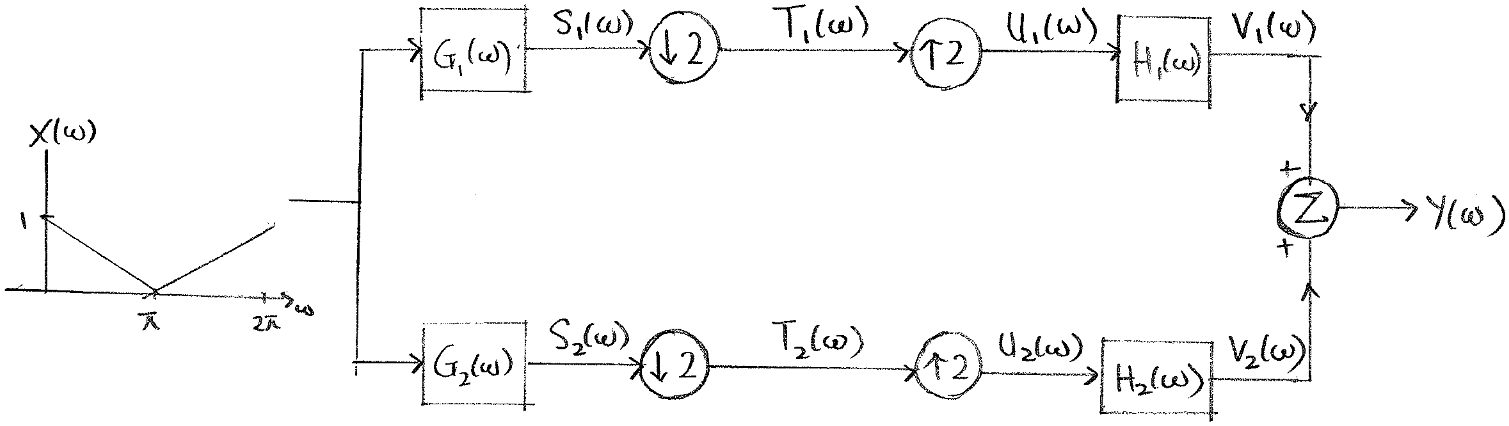


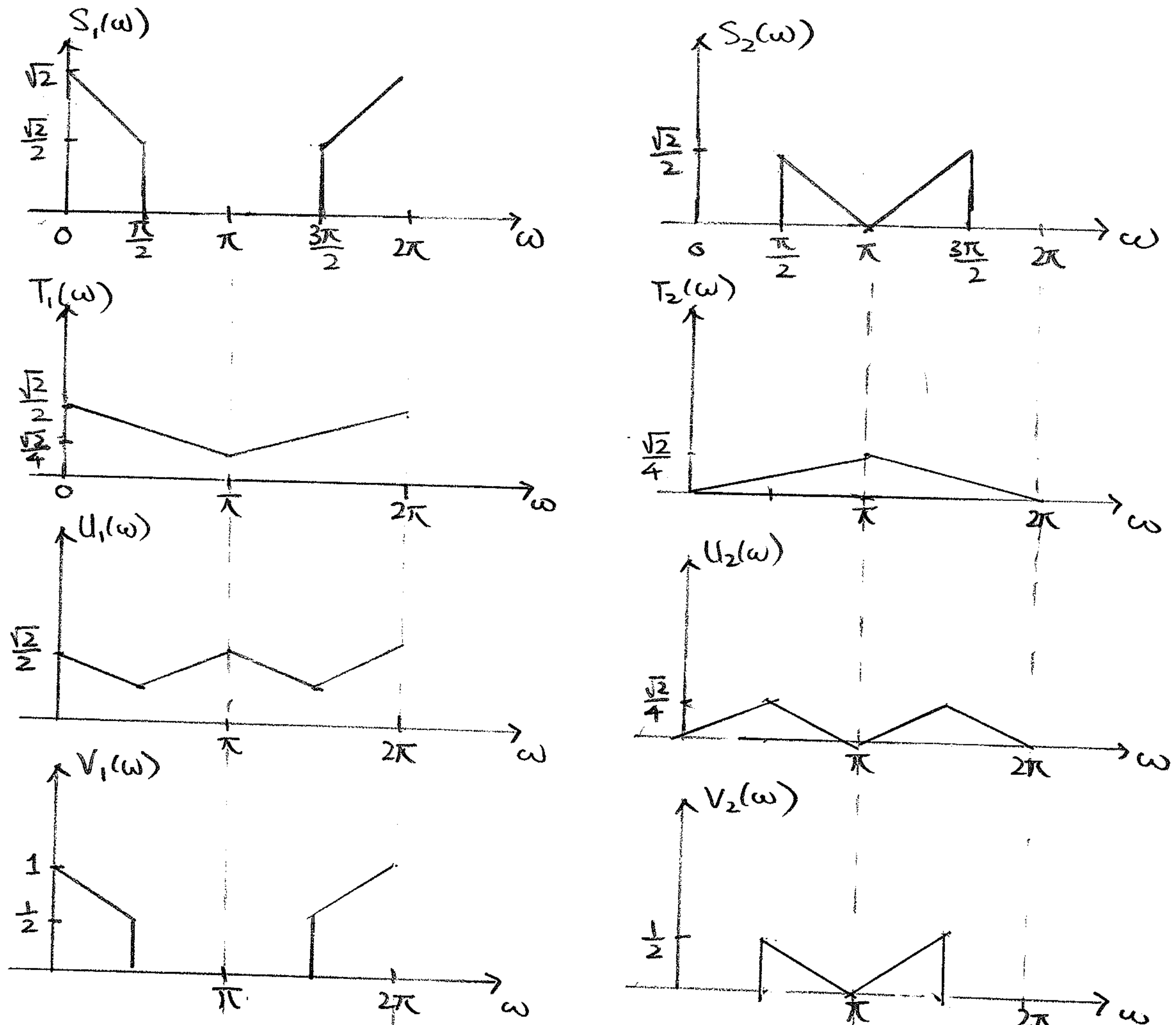
Suppose that the DTFT of the input to the system is given by:



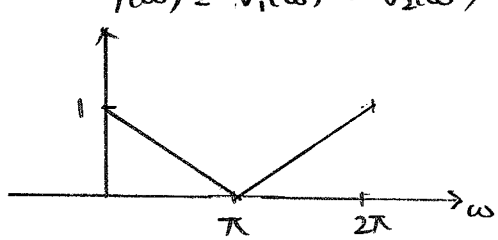
Carefully sketch the DTFT of the signal after each signal operation in both branches of the filter bank diagram shown above, including the DTFT of the final output. Since there are a total of nine operations, you will have nine spectral plots. You should label the points in the diagram either with numbers or letters; so that it will be clear where the DTFT spectra shown in your plots occur in the system. For purposes of this problem, assume that the blocks "Channel 1" and "Channel 2" do nothing to the signals that pass through them, and can thus be ignored.

3. (continued)





$Y(\omega) = V_1(\omega) + V_2(\omega)$



$Y(\omega) = X(\omega)$

perfect reconstruction

4. (25 pts.) Every Monday, Wednesday, and Friday, Lynn has a 10 minute break between her class that ends at 2:20p and ECE 438, which starts at 2:30p. Before going to ECE 438, she likes to stop by Beans in the MSEE atrium to get a cup of coffee. However, if there are too many people in line when she gets there, she has to forgo the coffee in order to not be late for class. The problem is to predict how long the wait will be if a given number of people are in line when she arrives at the atrium. On one day, there is only one person in line; and the wait is 1 minute. On another day, there are two people in line; and the wait is 3 minutes. Finally, on a third day, there are five people in line; and the wait is 8 minutes.

Lynn would like to model the predicted wait $\hat{W}$ as a linear function of the number of people P in line when she arrives at the atrium. So $\hat{W} = aP + b$, where a and b are constants.

a) (22) Find the least-squares solution for the parameters a and b that minimizes the mean-squared error between the prediction $\hat{W}$ and the three observations that Lynn has made.

b) (3) Plot your optimal predictor $\hat{W}$ in the $W - P$ plane along with the three data points.

(a)

P	W
1	1
2	3
5	8

$$\hat{W} = aP + b$$

$$E = \frac{1}{3}\sum_{D=1}^{3}\left[\hat{W}(P) - W(P)\right]^2$$

$$= \frac{1}{3}\left[(a+b-1)^2 + (2a+b-3)^2 + (5a+b-8)^2\right]$$

$$\frac{\partial E}{\partial a} = \frac{1}{3}\left\{2(a+b-1) + 2(2a+b-3)2 + 2(5a+b-8)5\right\}$$

$$= \frac{1}{3}\left\{2a+2b-2+8a+4b-12+50a+10b-80\right\}$$

$$= \frac{1}{3}\left\{60a+16b-94\right\} = 0$$

$$\Rightarrow 30a + 8b = 47$$

4. (continued)

$$\frac{\partial E}{\partial b} = \frac{1}{3}\{2(a+b-1) + 2(2a+b-3) + 2(5a+b-8)\}$$

$$= \frac{1}{3}\{2a+2b-2 + 4a+2b-6 + 10a+2b-16\}$$

$$= \frac{1}{3}\{16a+6b-24\} = 0$$

$$\Rightarrow 8a+3b = 12$$

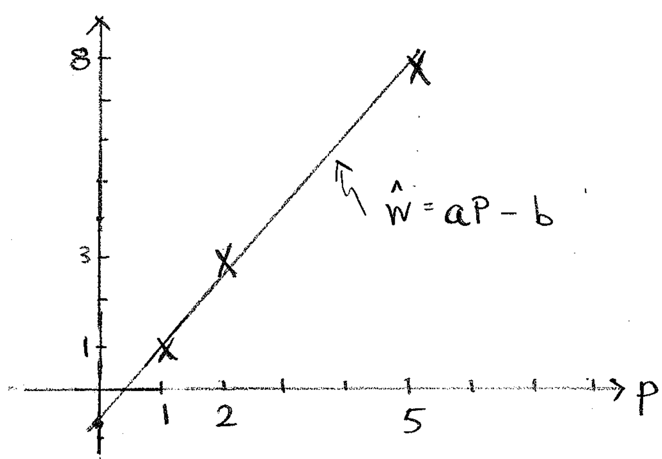
$$\begin{cases} 30a+8b = 47 & (*) \\ 8a+3b = 12 & (**) \end{cases}$$

$$3\cdot(*) - 8\cdot(**) \Rightarrow 26a = 45 \Rightarrow a = \frac{45}{26}$$

$$b = \frac{12-8a}{3} = \frac{12-\frac{360}{26}}{3} = \frac{\frac{-48}{26}}{3} = \frac{-24}{39}$$

$a = \frac{45}{26}$, $b = \frac{-24}{39}$ minimizes mean-squared error

(b)



1.	______
2.	______
3.	______
4.	______
Total	______