EE 438 Exam No. 3 Solutions Fall 1999

1. $E\{|x(t)|^2\} = \frac{1}{20}\int_{-10}^{10} x^2\,dx = \frac{1}{20}\left.\frac{x^3}{3}\right|_{-10}^{10}$

$= \frac{1000\cdot 2}{20\cdot 3} = \frac{100}{3} = 33.3$

$N = 2^{10}$

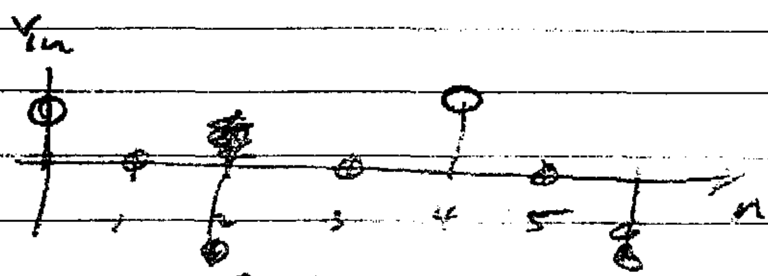
$\Delta = \frac{20}{N} = \frac{20}{2^{10}}$

$SNR = \frac{E\{|x(t)|^2\}}{E\{|e(t)|^2\}} = \frac{100/3}{\Delta^2/12} = \frac{400}{400}2^{20} = 2^{20}$

$SNR_{dB} = 10\log_{10}(SNR) = \cancel{\ldots}\,(20\log_{10}(2))$

$= 60$ dB

2. a.



b. $E\{Y_n\} = \begin{cases} E\{X_{n/2}\}, & n \text{ even} \\ E\{0\}, & n \text{ odd}\end{cases} = \begin{cases} 0, & n \text{ even} \\ 0, & n \text{ odd}\end{cases} = 0$

c. $E\{Y_m Y_n\} = \begin{cases} E\{Y_n^2\}, & m = n \\ E\{Y_m\}^{=0}E\{Y_n\}^{=0}, & m \neq n \text{ \& both even} \\ E\{Y_m 0\}, & m \text{ even}, n \text{ odd} \end{cases}$

$$\begin{cases} E\{0\, Y_n\} & m \text{ odd}, n \text{ even} \\ E\{0\,0\} & m \text{ odd}, n \text{ odd} \end{cases}$$

$$E\{Y_m Y_n\} = \begin{cases} E\{Y_n^2\}, & m = n \\ 0, & \text{else} \end{cases}$$

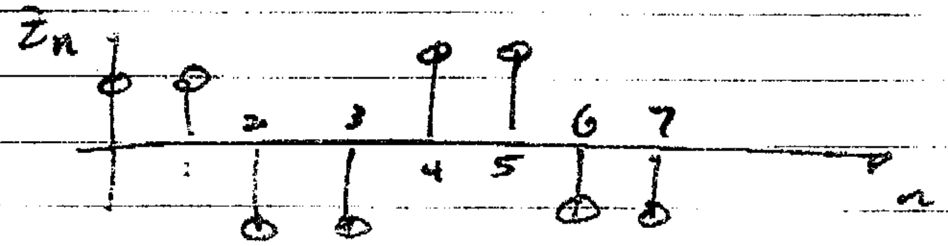
now $E\{Y_n^2\} = \begin{cases} E\{X_{n/2}\} ; & n \text{ even} \\ E\{0^2\} ; & n \text{ odd} \end{cases}$

$$= \begin{cases} 1, & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

So $E\{Y_m Y_n\} = \begin{cases} 1, & m = n \text{ \& even} \\ 0, & \text{else} \end{cases}$

d. No, $E\{Y_m Y_n\}$ is not a function of only $m-n$.

e.



f. $E\{Z_n\} = E\{\frac{1}{2}\{Y_n + Y_{n-1}\}\}$

$$= \frac{1}{2}\{E\{Y_n\} + E\{Y_{n-1}\}\}$$

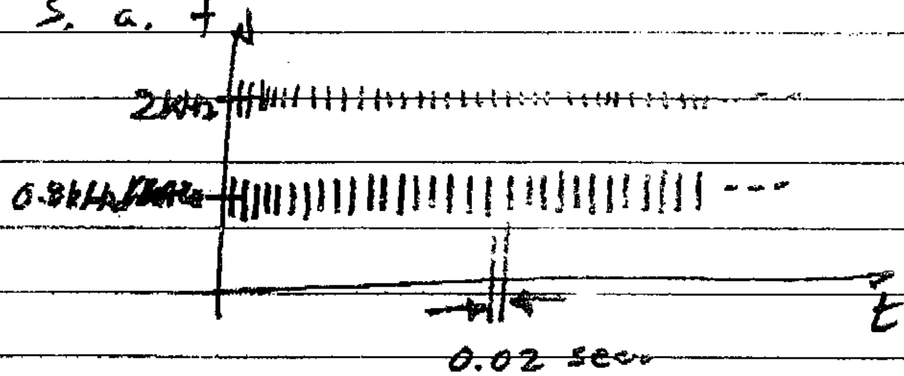
$$= \frac{1}{2}(0 + 0) = 0$$

g. $E\{Y_m Z_n\} = E\{Y_m \frac{1}{2}\{Y_n + Y_{n-1}\}\}$

$$= \frac{1}{2} E\{Y_m Y_n\} + \frac{1}{2} E\{Y_m Y_{n-1}\}$$

$$E\{Y_m Z_n\} = \begin{cases} \frac{1}{2}, & m = n \text{ \& both even} \\ \frac{1}{2}, & m = n-1 \text{ \& both even} \\ 0 : & \text{else} \end{cases}$$

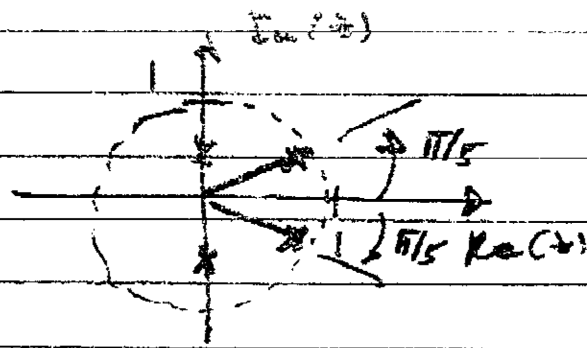
3. a.



b) i) $D = \dfrac{\frac{1}{50}}{\frac{1}{8000}} = \dfrac{800}{5} = 160$

ii) $\omega_1 = \dfrac{2\pi}{8000} \times 800 = \dfrac{2\pi}{10}$ rad./sample

$\omega_2 = \dfrac{2\pi}{8000} \times 2000 = \dfrac{\pi}{2}$ rad./sample



4. a. $\epsilon = \int_0^1 \left[(a+bt) - t^2\right]^2 dt$

$\dfrac{d\epsilon}{da} = \int_0^1 2\left[(a+bt) - t^2\right] \dfrac{da}{da}\, dt$ by chain rule

$= 2a + b - 2/3$

$$\frac{d\epsilon}{db} = \int_0^1 2\left[(a+bt) - t^2\right] t \, dt$$ again by chain rule

$$= 2a \Big|_0^1 \frac{t^2}{2} + 2b \Big|_0^1 \frac{t^3}{3} - 2 \Big|_0^1 \frac{t^4}{4}$$

$$= a + \frac{2}{3} b - \frac{1}{2}$$

set both derivatives to zero ⇒

$$\left.\begin{aligned} a + b/2 &= 1/3 \\ a + 2/3\, b &= 1/2 \end{aligned}\right\} -b/6 = -1/6 \Rightarrow b = 1$$

$$\Rightarrow a = 1/3 - 1/2 = -1/6$$

b.