September 22, 1999 **Name:** Solution (1/6)

EE 438 **Exam No. 1** **Fall 1999**

- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (30 pts.) Consider a filter (F) described by the following difference equation

$$y[n] = \tfrac{1}{2}(x[n] + x[n-2])$$

   a. Find the frequency response $F(e^{j\omega})$ for this filter.
   b. Find simple expressions for the magnitude $|F(e^{j\omega})|$ and phase $\underline{/F(e^{j\omega})}$ of the filter, and sketch them both.
   c. Describe in general terms the characteristics of this filter.

   Suppose we form a new DT system by cascading two of the filters F described above.

   d. Find a new difference equation that describes the overall system.
   e. Find an expression for the frequency reponse $H(e^{j\omega})$ for the overall system in terms of $F(e^{j\omega})$.

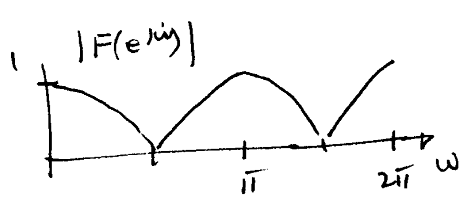
a) let $x[n] = e^{j\omega n}$

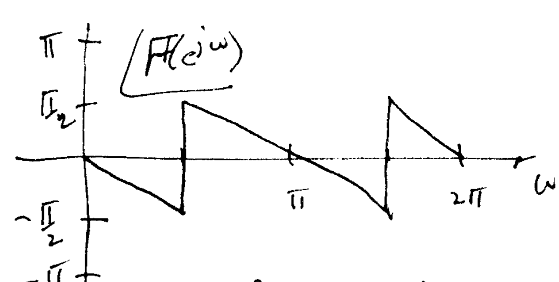
$$y[n] = F(e^{j\omega})\, x[n]$$
$$= \tfrac{1}{2}\left[e^{j\omega n} + e^{+j\omega(n-2)}\right]$$
$$= e^{-j\omega}\, \tfrac{1}{2}\left[e^{j\omega} + e^{-j\omega}\right] e^{j\omega n}$$

$\therefore \quad F(e^{j\omega}) = e^{-j\omega}\cos(\omega)$

b) $|F(e^{j\omega})| = |e^{-j\omega}\cos(\omega)| = |\cos(\omega)|$

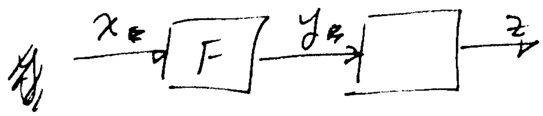
$$\angle F(e^{j\omega}) = \angle e^{-j\omega} + \angle \cos(\omega) = -\omega + \begin{cases} 0, & \cos(\omega) > 0 \\ \pm\pi, & \cos(\omega) < 0 \end{cases}$$





c) mid band stop (attenuates medium frequencies)

1. (continued)

d) 

$$z[n] = \frac{1}{2}\left(y[n] + y[n-2]\right)$$

$$y[n] = \frac{1}{2}\left(x[n] + x[n-2]\right)$$

$$\Rightarrow \quad z[n] = \frac{1}{2}\left\{ \frac{1}{2}\left(x[n] + x[n-2]\right) + \frac{1}{2}\left(x[n-2] + x(n-4)\right)\right\}$$

$$= \frac{1}{4}\left(x[n] + 2x[n-2] + x[n-4]\right)$$

e) let $x[n] = e^{j\omega n}$

$$y[n] = F(e^{j\omega})\, x[n]$$

$$z[n] = F(e^{j\omega})\, y[n]$$

$$= F^2(e^{j\omega})\, x[n]$$

$$\therefore \; H(e^{j\omega}) = F^2(e^{j\omega})$$

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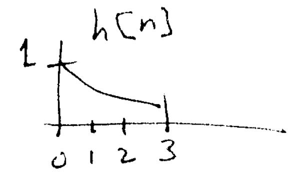
2. (30 pts.) Consider a DT LTI system with impulse response

$$h[n] = 2^{-n}(u[n] - u[n-4])$$

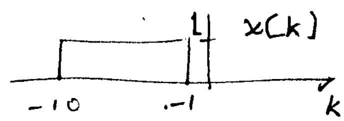
Find the ouput $y[n]$ of this system to the following two inputs $x[n]$:

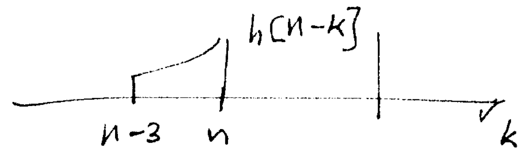
a. $x[n] = u[n+10] - u[n]$

b. $x[n] = \sum_{k=-\infty}^{\infty} (-1)^k \delta[n-10k]$



a) $y[n] = \sum_k h[n-k]x[k]$





Case I

$n < -10 \quad y[n] = 0$

case II

$-10 \le n$

$n-3 \le -10$

$-10 \le n \le -7$

$$y[n] = \sum_{k=-10}^{n} 2^{-(n-k)}$$

$$= 2^{-n} \sum_{k=-10}^{n} 2^{k} = 2^{-n} \sum_{\ell=0}^{n+10} 2^{\ell-10}$$

$\ell = k+10$

$$= 2^{-(n+10)} \sum_{\ell=0}^{n+10} 2^{\ell} = 2^{-(n+10)} \frac{1-2^{(n+11)}}{1-2}$$

$$= (2^{n+11} - 1)\, 2^{-(n+10)} = 2 - 2^{-(n+10)}$$

case III

$-7 < n \le -1$

$$y[n] = \sum_{k=n-3}^{n} 2^{-(n-k)} = 2^{-n} \sum_{k=n-3}^{n} 2^{k}$$

$\ell = k-(n-3)$

$$= 2^{-n} \sum_{\ell=0}^{3} 2^{\ell+(n-3)}$$

$$= 2^{-3} \sum_{\ell=0}^{3} 2^{\ell} = 2^{-3} \frac{1-2^{4}}{1-2} = 2 - 2^{-3}$$

$$= 1\tfrac{7}{8}$$

2. (continued)

case IV

$-1 < n$

$n-3 \le -1$

$-1 < n \le 2$

$$y[n] = \sum_{k=n-3}^{-1} 2^{-(n-k)} = 2^{-n} \sum_{k=n-3}^{-1} 2^{k} = 2^{-n} \sum_{l=0}^{-n+2} 2^{l+n-3}$$

$l = k-(n-3)$

$$= 2^{-3} \frac{1-2^{-n+3}}{1-2} = 2^{-3}\left(2^{-n+3}-1\right)$$

case V

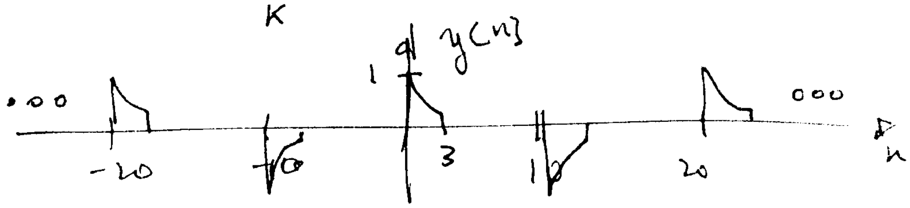
$2 < n$ $\qquad y[n] = 0$

b) $$y[n] = \sum_{l} h[n-l]\, x[l]$$

$$= \sum_{l} h[n-l] \sum_{k} (-1)^{k} \delta[l-10k]$$

$$= \sum_{k} (-1)^{k} \sum_{l} h[n-l]\, \delta[l-10k]$$

$$= \sum_{k} (-1)^{k} h[n-10k]$$



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3 (20 pts.) Consider an ideal sampler operating at a sampling frequency of 8 kHz. The input to the sampler is the (analog) CT signal $x_a(t)$. The output is the (digital) DT signal $x_d[n] = x_a(nT)$, where T is the sampling interval.

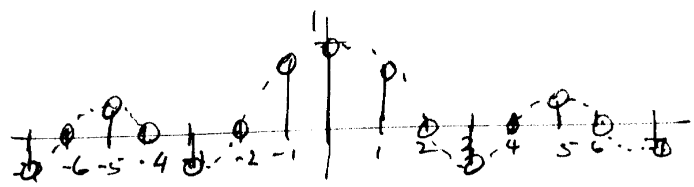
Assume that $x_a(t) = \sin c(4000t)$

a. Carefully sketch a plot of $x_d[n]$ for $-7 \le n \le 7$. Be sure to fully dimension your sketch.

b. Find the DTFT $X_d(e^{j\omega})$ of $x_d[n]$ by using the relationship between the CTFT $X_a(f)$ of $x_a(t)$ and the DTFT $X_d(e^{j\omega})$.

c. Plot the DTFT $X_d(e^{j\omega})$. Be sure to fully dimension your sketch.

a) $x_d[n] = x_a(nT) = \operatorname{sinc}\left(4000 \frac{n}{8000}\right) = \operatorname{sinc}\left(\frac{n}{2}\right)$



b) let $x_s(t) = \operatorname{comb}_T[x_a(t)]$

$$X_s(f) = \frac{1}{T} \operatorname{rep}_{\frac{1}{T}} [X(f)]$$

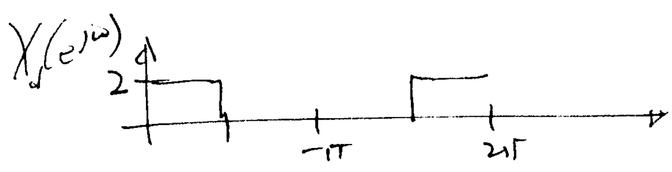
$$= 8000 \operatorname{rep}_{8000}\left[\frac{1}{4000} \operatorname{rect}\left(\frac{f}{4000}\right)\right]$$

$$= 2 \sum_k \operatorname{rect}\left(\frac{f - 8000k}{4000}\right)$$

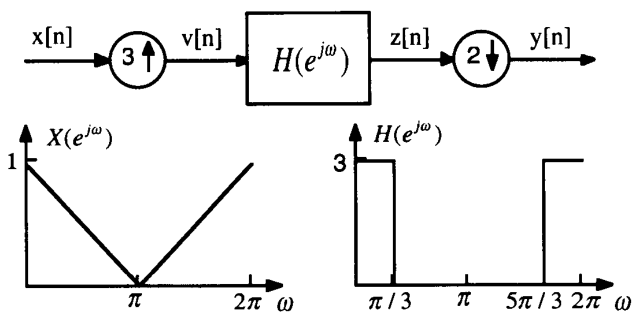
$$X_d(e^{j\omega}) = X_s(f)\Big|_{f = \frac{\omega}{2\pi T}}$$

$$= 2 \sum_k \operatorname{rect}\left(\left(\frac{\omega}{2\pi} - k\right)2\right)$$

$$= 2 \sum_k \operatorname{rect}\left(\frac{\omega - 2\pi k}{\pi}\right)$$



4. (20 pts.) Consider the DT system shown below



Carefully sketch the following spectra being sure to dimension your plots:

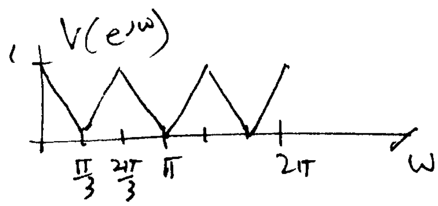
a. $V(e^{j\omega})$

b. $Z(e^{j\omega})$

c. $Y(e^{j\omega})$

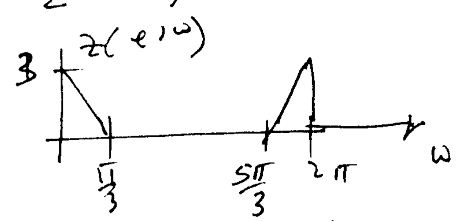
d. Describe the overall operation of this system, i.e. what does it do?

a) $V(e^{j\omega}) = X(e^{j\omega 3})$

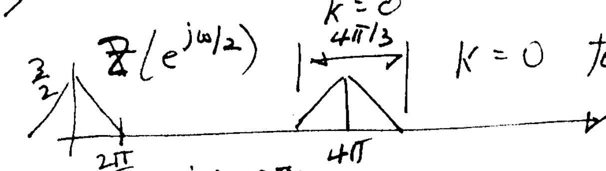


d) Comparing $X(e^{j\omega})$ (Nyquist sampled) and $Y(e^{j\omega})$ (oversampled), we see that sampling rate has been increased by 1.5x

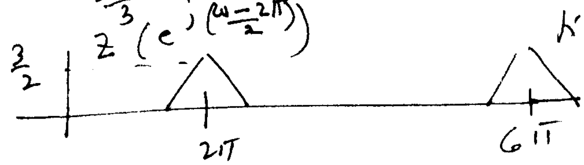
b) $Z(e^{j\omega}) = V(e^{j\omega}) H(e^{j\omega})$



c) $Y(e^{j\omega}) = \frac{1}{2} \sum_{k=0}^{1} Z\left(e^{j\frac{\omega-2\pi k}{2}}\right)$



k = 0 term



k = 1 term

add

$Y(e^{j\omega})$