November 23, 1998 **Name:**_____

EE 438 **Exam No. 3** **Fall 1998**

- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (25 pts.) Consider the random process defined by

$$X[n] = 2\cos\left(\tfrac{\pi}{2}n + \theta\right)$$

where θ is a random variable uniformly distributed on the interval $[0, 2\pi)$

a. (8) Find the expected value of X[n].

b. (8) Find the variance of X[n].

c. (8) Find the autocorrelation $r_{XX}[m,n] = E\{X[m]X[n]\}$ of this process.

d. (1) Is X[n] wide-sense stationary?

Note: the following formula may be useful.

$$\cos\alpha\,\cos\beta = \tfrac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

1. (continued)

2. (25 pts.) A wide-sense stationary, unit mean random process X[n] has autocorrelation

$$r_{XX}[n] = \begin{cases} 2, & n = 0 \\ 1.5, & n = \pm 1 \\ 1, & \text{else} \end{cases}$$

The process is passed through a first order discrete-time differentiator, described by the filter equation

$$Y[n] = X[n] - X[n-1]$$

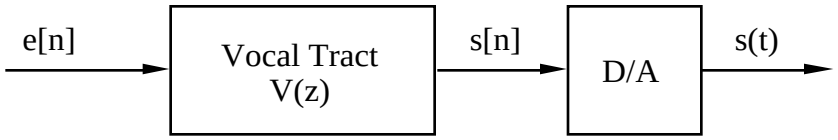
a. (4) Find the mean E{Y[n]} of the output process

b. (20) Find the autocorrelation $r_{YY}[n]$ of the output process.

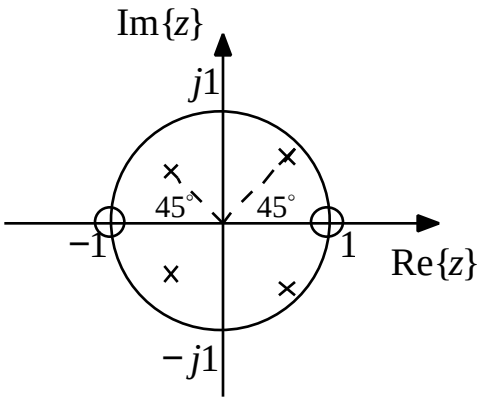
c. (1) How does the differentiator affect the “correlateness” of the input process?

2. (continued)

3. (25 pts.) The digital synthesizer for voiced speech shown below operates at a 10 kHz sampling rate.



The excitation is given by $e[n] = \sum_{k=-\infty}^{\infty} \delta[n - 200k]$. The vocal tract transfer function $V(z)$ has poles and zeros at the locations shown below:



a. (10) What is the pitch period in seconds?

b. (10) Find the formant frequencies of s(t) in analog units of Hz, and rank them according to their strength, *i.e.* how peaked the vocal tract response is at the corresponding frequency.

c. (5) Sketch roughly what the continuous-time Fourier transform (CTFT) of the waveform s(t) would look like.

3. (continued)

4. (25 pts) Consider the continuous-time (CT) signal x(t) below:

$$x(t) = [1 + \cos(2\pi 100t)]\cos(2\pi 5000t)$$

a. (8) Sketch x(t).

Consider x(t) as a model for a speech waveform

b. (1) Does it represent a voiced or unvoiced phoneme?

c. (3) What is the pitch period?

d. (3) How many formants are there, and at which frequencies do they occur?

Define a CT version of the short time Fourier transform (STCTFT) as

$$X(f,t) = \int_{-\infty}^{\infty} x(s)w(t-s)e^{-j2\pi fs}ds$$

where w(t) is the window function that time-limits the Fourier transform.

e. (10) Assume that w(t) = rect(t/0.1). Find the STCTFT of the signal $x(t) = e^{j2\pi f_0 t}$.

4. (continued)

1. _____

2. _____

3. _____

4. _____

Total _____