

- You have 50 minutes to work the following four problems.
 - Be sure to show all your work to obtain full credit.
 - The exam is closed book and closed notes.
 - Calculators are permitted.
1. (25 pts.) Consider the causal LTI system described by the difference equation
- $$y[n] = x[n] + y[n - 1]$$
- a. (14) Use the ZT to find the impulse response $h[n]$ for this system.
 - b. (10) Use the ZT to find the response $y[n]$ of the system to the input $x[n] = u[n]$.
 - c. (1) Is this system BIBO stable?

1. (continued)

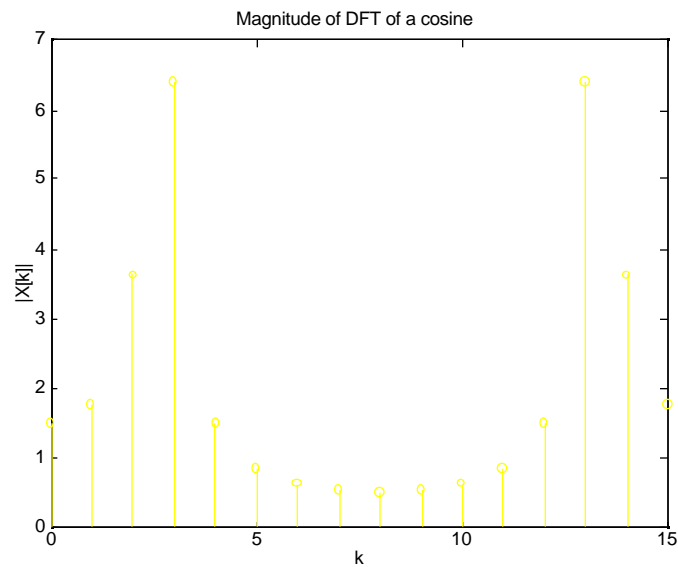
2. (25 pts.) Let $x[n]$, $n = 0, \dots, N-1$ be an N point signal with N point DFT $X^N[k]$, $k = 0, \dots, N-1$. Now generate a new $2N$ point signal $y[n]$, $n = 0, \dots, 2N-1$ by simply repeating $x[n]$, i.e.

$$y[n] = \begin{cases} x[n], & 0 \leq n \leq N-1 \\ x[n-N], & N \leq n \leq 2N-1 \end{cases}$$

Find a simple expression for the $2N$ point DFT $Y^{2N}[k]$, $k = 0, \dots, 2N-1$ in terms of $X^N[k]$, $k = 0, \dots, N-1$.

2. (continued)

3. (25 pts.) The plot below shows the magnitude of the DFT of a cosine $x[n]$ which was obtained by sampling an analog waveform at a 10kHz rate.
- (10) Estimate the digital frequency ω_d in radians/sample of this signal.
 - (5) Assuming that the analog waveform was not undersampled, what is the corresponding analog frequency f_a ?
 - (5) Find the digital frequency ω_d closest to ω_d for which there is no leakage or picket fence effect.
 - (5) Sketch approximately what the magnitude of the DFT would look like in this case.



3. (continued)

4. (25 pts.) Derive the decimation in time fast Fourier transform algorithm for a 6 point DFT. Show a complete flow diagram, including all twiddle factors for your FFT.

4. (continued)

1. _____

2. _____

3. _____

4. _____

Total _____