26 September 2018 **Name:___________________**

ECE 438 **Exam No. 1** **Fall 2018**

- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are **not** permitted.

1. (25 pts.) Consider the linear, time-invariant system defined by the difference equation

$$y[n] = \frac{1}{3}\left\{x[n] + x[n-1] + x[n-2]\right\}.$$

a. (9) Find the response of this system to the input

$$x[n] = \begin{cases} 1, & 0 \le n \le 2 \\ 0, & \text{else} \end{cases}.$$

b. (9) Find a simple expression for the frequency response $H(\omega)$ of this system.

c. (7) Based on your answer to part (b), find a simple expression for the magnitude $|H(\omega)|$ of the frequency response of this system.

1. (continued)

(9) a. $y[n] = \frac{1}{3}\{\delta[n] + \delta[n-1] + \delta[n-2] +$

$\delta[n-1] + \delta[n-2] + \delta[n-3] +$

$\delta[n-2] + \delta[n-3] + \delta[n-4]\}$

$= \frac{1}{3}\{\delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4]\}$ +4

$$y[n] = \begin{cases} \frac{1}{3}, & n = 0, 4 \quad +2 \\ \frac{2}{3}, & n = 1, 3 \quad +2 \\ 1, & n = 2. \quad +1 \\ 0, & \text{elsewhere} \end{cases}$$

(If all answers are right, make 9 points; otherwise, the points equals to the number of right answers at each point of (0, 1, 2, 3, 4, 5))

(9) b. $h[n] = \frac{1}{3}\{\delta[n] + \delta[n-1] + \delta[n-2]\}$

$H(w) = \frac{1}{3} + \frac{1}{3}e^{-jw} + \frac{1}{3}e^{-j2w}$

$= \frac{1}{3}e^{-jw}(e^{jw} + 1 + e^{-jw})$ +7

$= \frac{1}{3}e^{-jw}(1 + 2\cos w)$ +2 (final step)

(7) c. $|H(w)| = |\frac{1}{3}e^{-jw}(1 + 2\cos w)|$ (absolute sign). +2.

$= \frac{1}{3}|e^{-jw}||1 + 2\cos w|$

$= \frac{1}{3}|1 + 2\cos w|$ +5

2. (30 pts.) Perform the convolution $w[n]$ of the following two signals, and carefully sketch the output signal $w[n]$. Your solution for $w[n]$ should be an analytical expression or expression(s) for the signal.

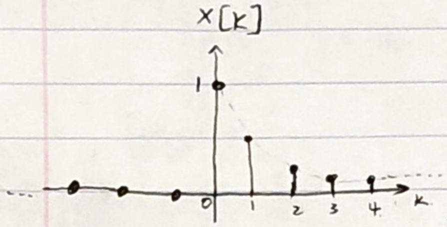
$$x[n] = \left(\frac{1}{2}\right)^n u[n], \text{ and } y[n] = \begin{cases} 1, & |n| \le 9 \\ 0, & \text{else} \end{cases}.$$

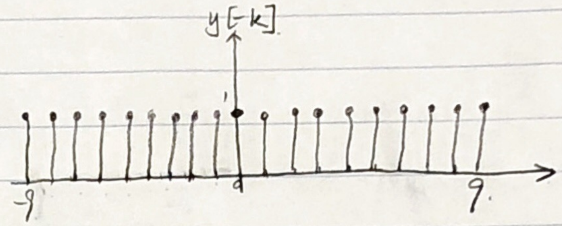
ECE 438 Exam 1 Problem2

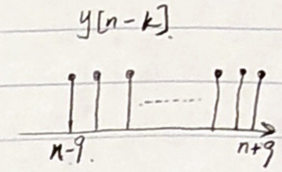
Problem 2. Perform $w[n]$ and sketch $w[n]$ $w[n] = x[n] * y[n]$

$x[n] = (\frac{1}{2})^n u[n]$, $y[n] = \begin{cases} 1, & |n| \le 9 \\ 0, & \text{else} \end{cases}$

Solve: $w[n] = x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]\, y[n-k]$







case 1: $n+9<0$ or $n<-9$, $w[n]=0$ $\boxed{5}$

case 2: $\begin{cases} n+9 \ge 0 \\ n-9 \le 0 \end{cases}$, or $\begin{cases} n \ge -9 \\ n \le 9 \end{cases}$, $-9 \le n \le 9$ $\boxed{2}$

$w[n] = \sum_{k=0}^{n+9} (\frac{1}{2})^k$ $\boxed{3}$ $= \frac{1-(\frac{1}{2})^{n+9+1}}{1-\frac{1}{2}}$ $\boxed{3}$ $= \frac{1-(\frac{1}{2})^{n+10}}{1-\frac{1}{2}} = 2 - (\frac{1}{2})^{n+9}$ $\boxed{2}$

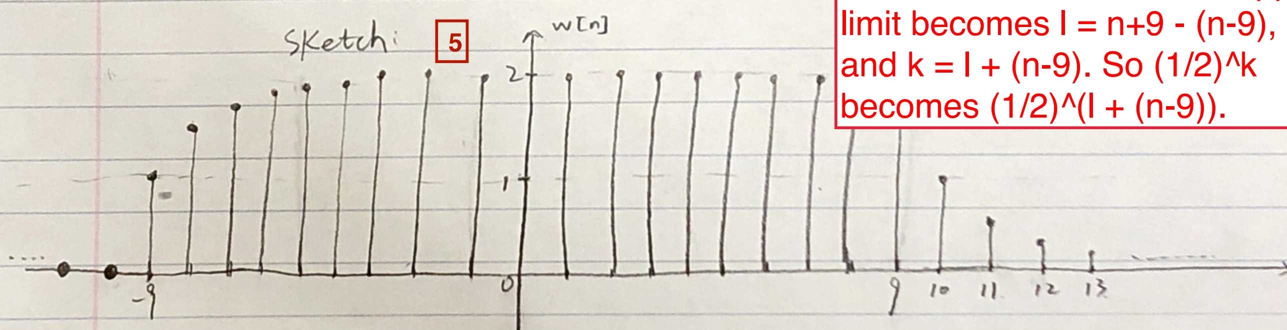
case 3: $n-9>0$ or $n>9$ $\boxed{2}$

$w[n] = \sum_{k=n-9}^{n+9} (\frac{1}{2})^k$ $\boxed{3}$ $= (\frac{1}{2})^{n-9} \cdot \frac{1-(\frac{1}{2})^{[n+9-(n-9)+1]}}{1-\frac{1}{2}}$ $\boxed{3}$ $= (\frac{1}{2})^{n-9} \cdot \frac{1-(\frac{1}{2})^{19}}{\frac{1}{2}} = (\frac{1}{2})^{n-10} - (\frac{1}{2})^{n+9}$ $\boxed{2}$

It would be good to add an extra step here, where you explicitly show the change of index of summation: let l = k-(n-9), then lower limit becomes l = 0, upper limit becomes l = n+9 - (n-9), and k = l + (n-9). So (1/2)^k becomes (1/2)^(l + (n-9)).

$$\Rightarrow w[n] = \begin{cases} 2-(\frac{1}{2})^{n+9}, & -9 \le n \le 9 \\ (\frac{1}{2})^{n-10} - (\frac{1}{2})^{n+9}, & n > 9 \\ 0, & \text{o.w.} \end{cases}$$

Sketch: $\boxed{5}$



Calculate by matlab

n	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
$w[n] \approx$	0	1	1.5	1.75	1.875	1.93	1.97	1.98	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	2	2	2	2
n	10	11	12	13	....															
$w[n] \approx$	1	0.5	0.25	0.125	....															

3. (20) Consider the continuous-time signal $x(t)=\cos\left(2\pi(4000)t\right)+\cos\left(2\pi(8000)t\right)$.

a. (6) Find a simple expression for the CTFT $X(f)$ of $x(t)$, and sketch it.

Suppose that we sample $x(t)$ at a 10 kHz sampling rate to generate the continuous-time sampled signal $x_s(t)=\mathrm{comb}_T\left[x(t)\right]$, where $T=1/10000$ sec. is the sampling interval.

b. (8) Find a simple expression for the CTFT $X_s(f)$ of $x_s(t)$, and sketch it. Your final answer should not contain any operators.

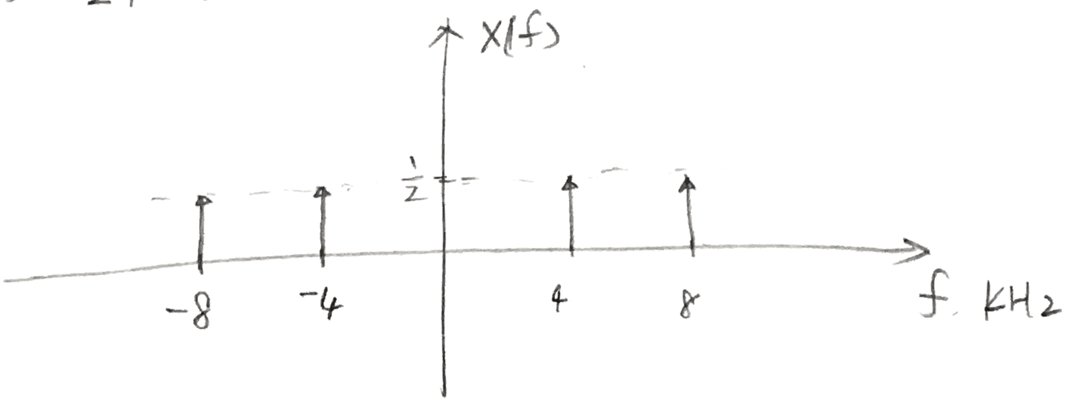
Now, suppose that we input $x_s(t)$ to an ideal low-pass reconstruction filter with frequency response $H_r(f)=T\cdot\mathrm{rect}(Tf)$.

c. (6) Find a simple expression for the output $x_r(t)$ from the reconstruction filter. Your final answer should not contain any operators.

a) $\cos(2\pi f_0 t) \xleftrightarrow{CTFT} \frac{1}{2}\{\delta(f-f_0)+\delta(f+f_0)\}$

thus.

$X(f) = \frac{1}{2}\{\delta(f-4000)+\delta(f+4000)\} + \frac{1}{2}\{\delta(f-8000)+\delta(f+8000)\}$



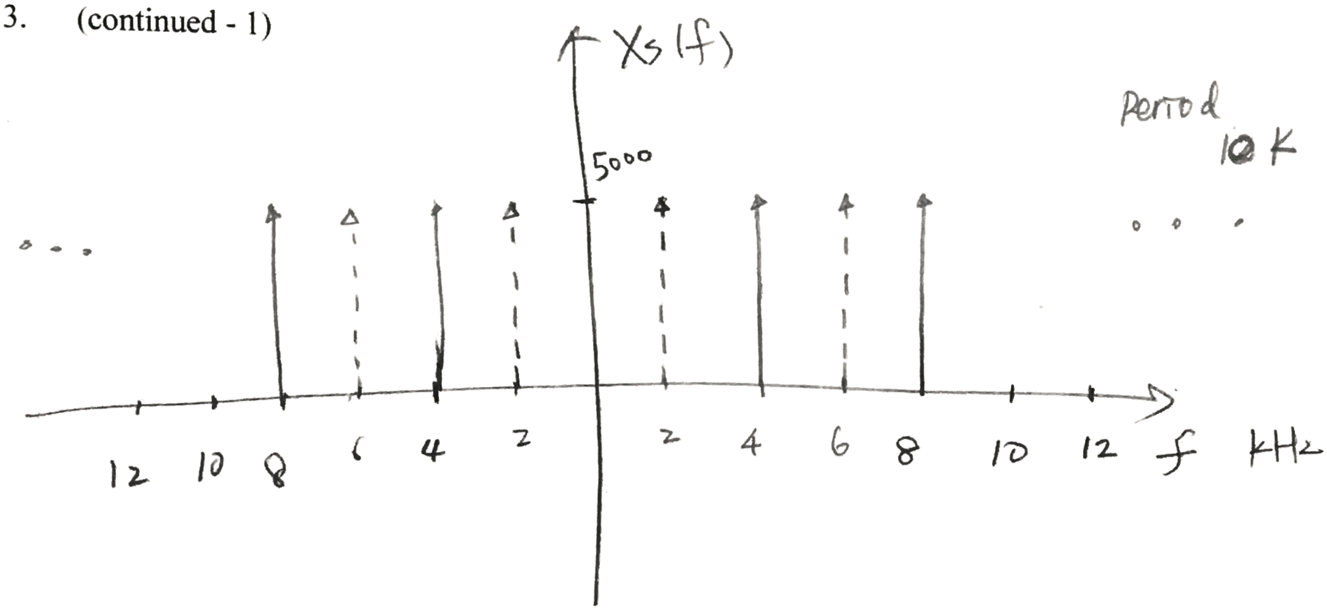
b) $\mathrm{comb}_T[x(t)] \xleftrightarrow{CTFT} \frac{1}{T}\,\mathrm{rep}_{\frac{1}{T}}[X(f)]$

thus. $X_s(f) = \frac{1}{T}\,\mathrm{rep}_{\frac{1}{T}}[X(f)]$

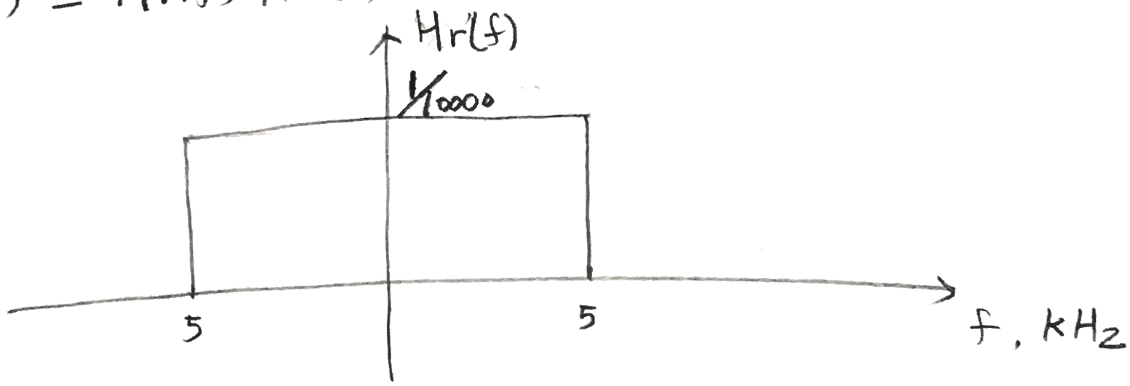
$= 10000\,\mathrm{rep}_{10000}\left\{\frac{1}{2}\{\delta(f-4000)+\delta(f+4000)+\delta(f-8000)+\delta(f+8000)\}\right\}$

$= 5000\,\mathrm{rep}_{10000}\{\delta(f-4000)+\delta(f+4000)+\delta(f-8000)+\delta(f+8000)$

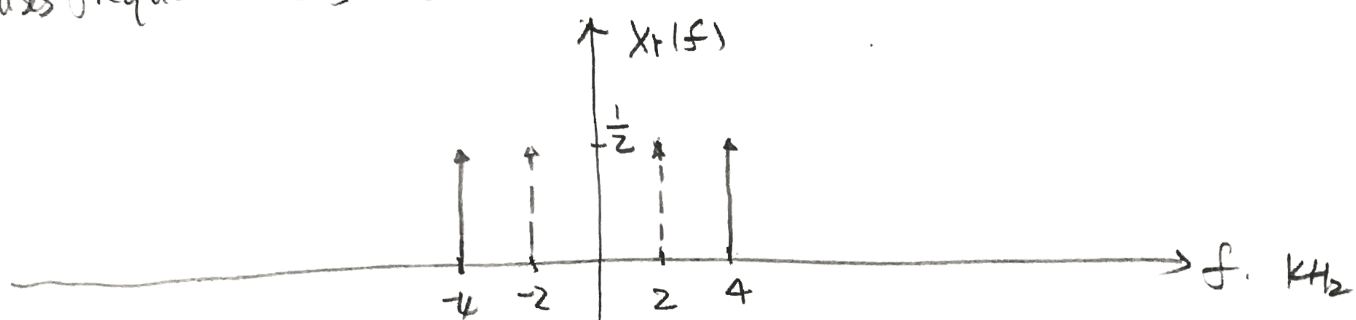
3. (continued - 1)



c) $X_r(f) = H_r(f)\, X_s(f)$



After the low pass filter, only base band survive, and impulses frequence < 5 kHz survives.

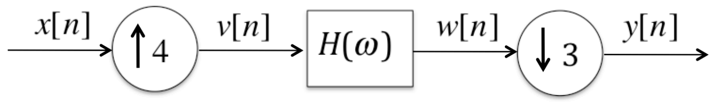


$X_r(f) = H_r(f) \cdot X_s(f) = \frac{1}{2}\delta(f-2000) + \frac{1}{2}\delta(f+2000) + \frac{1}{2}\delta(f-4000) + \frac{1}{2}\delta(f+4000)$

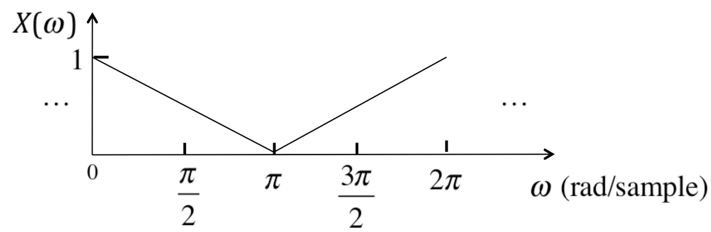
Inverse CTFT

$x_r(t) = \cos(2\pi(2000t)) + \cos(2\pi(4000)t)$

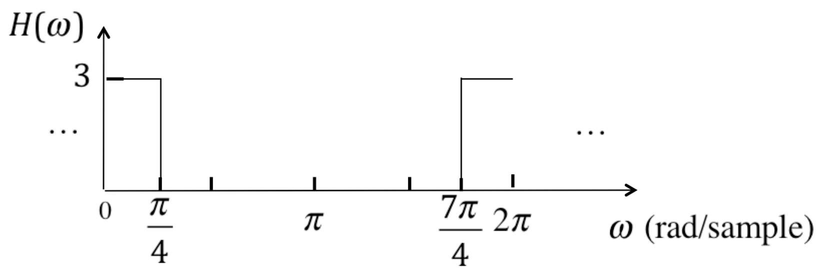
4. (25 pts) Consider the system shown below:



The DTFT of the input is given by:



The frequency response of the filter is given by:



a) (18) Carefully sketch the DTFTs $V(\omega)$, $W(\omega)$, $Y(\omega)$.

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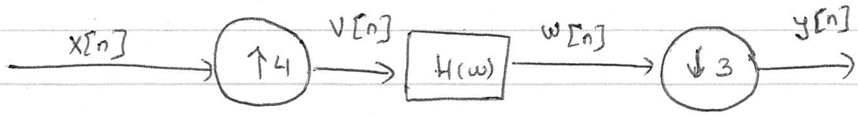
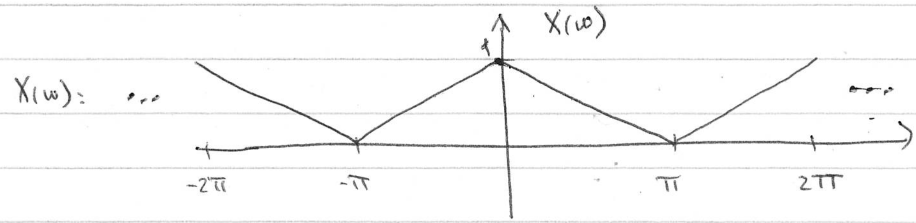
b) (3) Describe the overall effect that this system has on its input.

c) (4) For the given $X(\omega)$, would the system work, if we first downsampled by $3\times$, filtered with a lowpass filter, and then upsampled by $4\times$? Why or why not?

Question 4



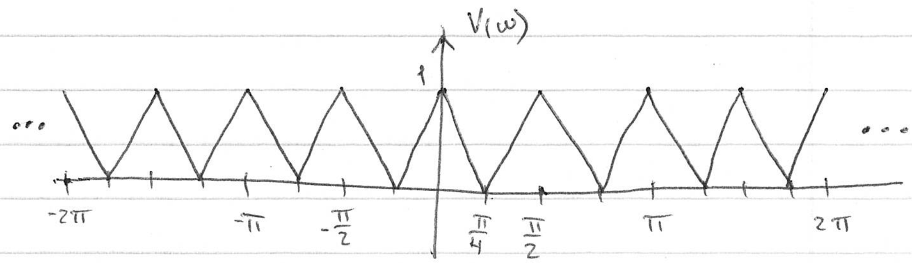


$V(\omega) = X(4\omega)$

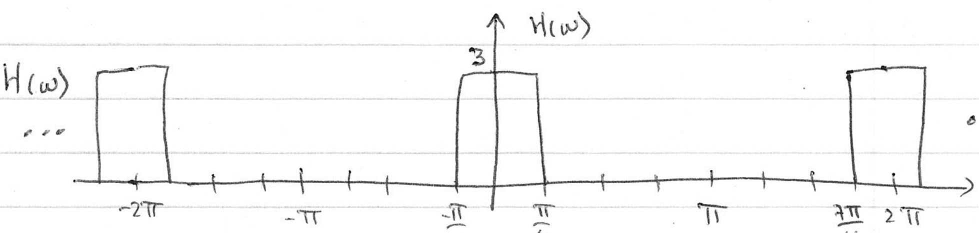
$V(0) = X(0)$

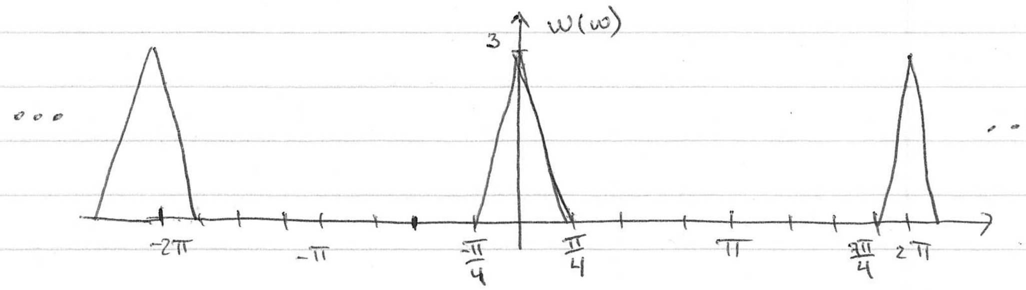
$V(\frac{\pi}{4}) = X(\pi)$

$V(\frac{\pi}{2}) = X(2\pi)$

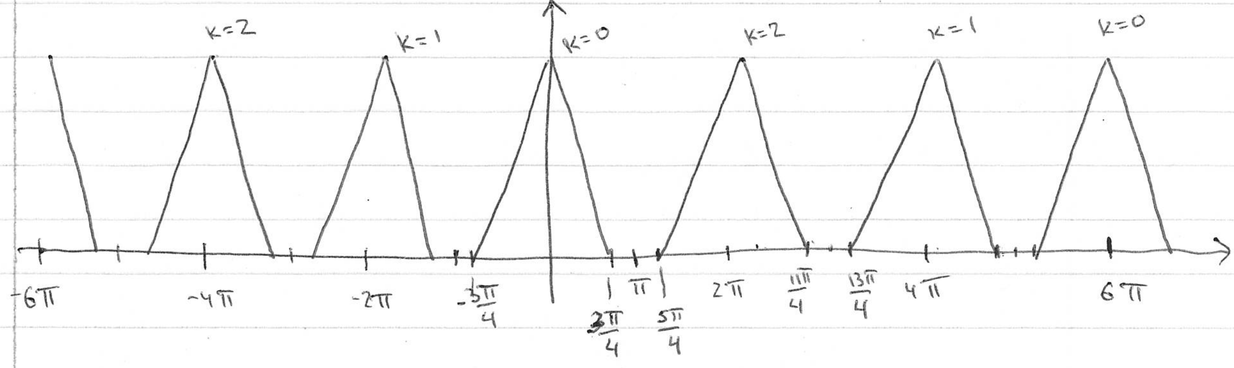


$W(\omega) = V(\omega) \cdot H(\omega)$





$$Y(\omega) = \frac{1}{3}\sum_{k=0}^{2} W\left(\frac{\omega + 2\pi k}{3}\right) = \frac{1}{3}\left\{ W(\omega/3) + W\left(\frac{\omega+2\pi}{3}\right) + W\left(\frac{\omega+4\pi}{3}\right)\right\}$$



b) This system changes sampling frequency of the input signal

$$f_{s_{new}} = \frac{4}{3} f_s$$

c) No, the system wouldn't work.

If we try to downsample the given signal $X(\omega)$ by 3 times, there would be aliasing, because the signal wasn't prefiltered