

- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are **not** permitted.

1. (25 pts.) Consider a 3-level quantizer defined as

$$Q(x) = \begin{cases} -1, & -\frac{3}{2} \leq x < -\frac{1}{2} \\ 0, & -\frac{1}{2} \leq x < \frac{1}{2} \\ 1, & \frac{1}{2} \leq x \leq \frac{3}{2} \end{cases} \quad (1)$$

- a. (8) Suppose that the input to this quantizer is the waveform $x(t) = \sin(2\pi t / 12)$ where the units of time t are seconds. Carefully sketch the waveforms $x(t)$ and $y(t) = Q(x(t))$ on the same axes for the interval $0 \leq t \leq 1$. Be sure to dimension both axes of your plot.
- b. (4) Using the formula derived in class that is based on the assumption that the quantizer error is uniformly distributed over the quantizer step-size, calculate the mean-squared quantization error for the quantizer given by Eq. (1) above. For this problem, you should assume that the range of the input signal is from $-\frac{3}{2}$ to $\frac{3}{2}$.
- c. (13) Now suppose that the input to the quantizer given by Eq. (1) is a random variable X with first order probability density function

$$f_X(x) = \begin{cases} |x|, & |x| < 1 \\ 0, & \text{else} \end{cases} \quad (2)$$

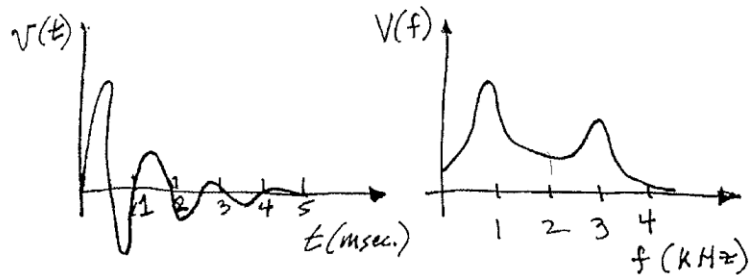
Compute the exact mean-squared quantization error $e_{\text{ms}} = E \{ [Q(X) - X]^2 \}$ for the quantizer given by Eq. (1) and the input with density given by Eq. (2).

2. (25 pts.) Consider four random variables X_0 , X_1 , X_2 , and X_3 that are independent and identically distributed, each with mean 2 and variance 1. Now suppose that we define three new random variables

$$\begin{aligned} Y_0 &= \frac{1}{2}(X_0 + X_1) \\ Y_1 &= \frac{1}{2}(X_1 + X_2) \\ Y_2 &= \frac{1}{2}(X_2 + X_3) \end{aligned} \tag{1}$$

- a. (3) Compute the expected value (mean) of each of the random variables Y_0 , Y_1 , and Y_2 .
- b. (6) Compute the variance of each of the random variables Y_0 , Y_1 , and Y_2 .
- c. (8) Compute the correlation $E\{Y_0 Y_1\}$, covariance $\sigma_{Y_0 Y_1}^2 = E\{(Y_0 - \bar{Y}_0)(Y_1 - \bar{Y}_1)\}$, and correlation coefficient $\rho_{Y_0 Y_1}$ for the two random variables Y_0 and Y_1 .
- d. (6) Compute the correlation $E\{Y_0 Y_2\}$, covariance $\sigma_{Y_0 Y_2}^2 = E\{(Y_0 - \bar{Y}_0)(Y_2 - \bar{Y}_2)\}$, and correlation coefficient $\rho_{Y_0 Y_2}$ for the two random variables Y_0 and Y_2 .
- e. (2) Comment on the relationship between your answers to parts (c) and (d) above. Specifically, what does this have to do with filtering of random sequences, and the relation between the autocorrelation function of the input to the filter and the autocorrelation function of the output of the filter?

3. (25) Consider a voiced phoneme for which the time-domain continuous-time vocal tract response $v(t)$ and corresponding frequency response (CTFT) $V(f)$ are given below.



- a. (5) Assume that the pitch frequency for the speaker is 200 Hz. Sketch what the continuous-time domain speech waveform $s(t)$ would look like in this case. Be sure to dimension all important quantities in the speech waveform.
- b. (10) Suppose that we sample the speech waveform $s(t)$ above at a 10 kHz rate, and compute the short-time discrete-time Fourier transform (STDFT) using a window of length 25 samples. Carefully sketch the resulting spectrogram. Be sure to dimension all important quantities in your plot. Is this a wide-band or narrow-band spectrogram?
- c. (10) Suppose that we sample the speech waveform $s(t)$ above at a 10 kHz rate, and compute the short-time discrete-time Fourier transform (STDFT) using a window of length 1,000 samples. Carefully sketch the resulting spectrogram. Be sure to dimension all important quantities in your plot. Is this a wide-band or narrow-band spectrogram?

4. (25 pts) Consider the signal $x(t) = t^2$ that is defined over the interval $0 \leq t \leq 1$. We wish to approximate $x(t)$ over this interval by $\hat{x}(t) = a_0 s_0(t) + a_1 s_1(t)$, where the basis functions $s_0(t)$ and $s_1(t)$ are given by

$$\begin{aligned} s_0(t) &= 1, \\ s_1(t) &= t, \end{aligned} \quad 0 \leq t \leq 1, \quad (1)$$

and the terms a_0 and a_1 are constants.

- a. (18) Determine the values for a_0 and a_1 that minimize the mean-squared approximation error

$$e_{\text{ms}} = \int_0^1 [\hat{x}(t) - x(t)]^2 dt.$$

- b. (7) Using your values for a_0 and a_1 determined in part (a) above, carefully sketch $x(t)$ and $\hat{x}(t)$ on the same axes for the interval $0 \leq t \leq 1$. Be sure to label and dimension both axes.

