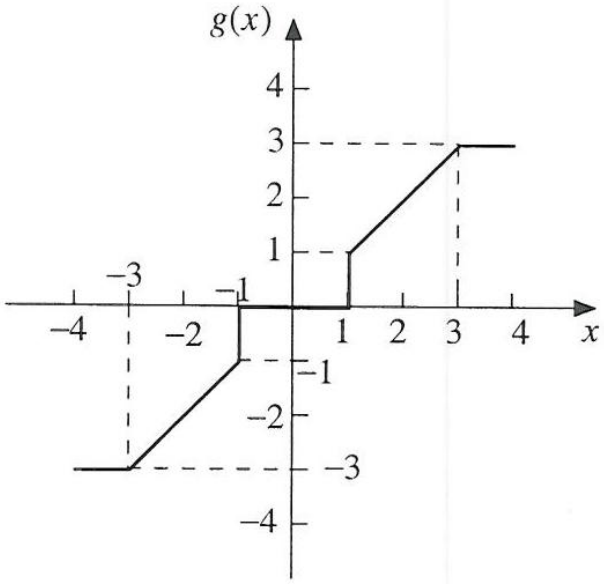
- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (25 pts.) Let X be a random variable that is uniformly distributed on the interval $[-4,4]$. Consider the function shown below:

Y = g(



Define a new random variable Y according to $Y = g(X)$.

a. (8) Find the mean and variance for Y

b. (12) Find the correlation coefficient ρ_{XY}

c. (5) Find the density function $f_Y(y)$.

2. (25 pts.) Consider the spectrogram and waveform shown below for the utterance "Rice University".

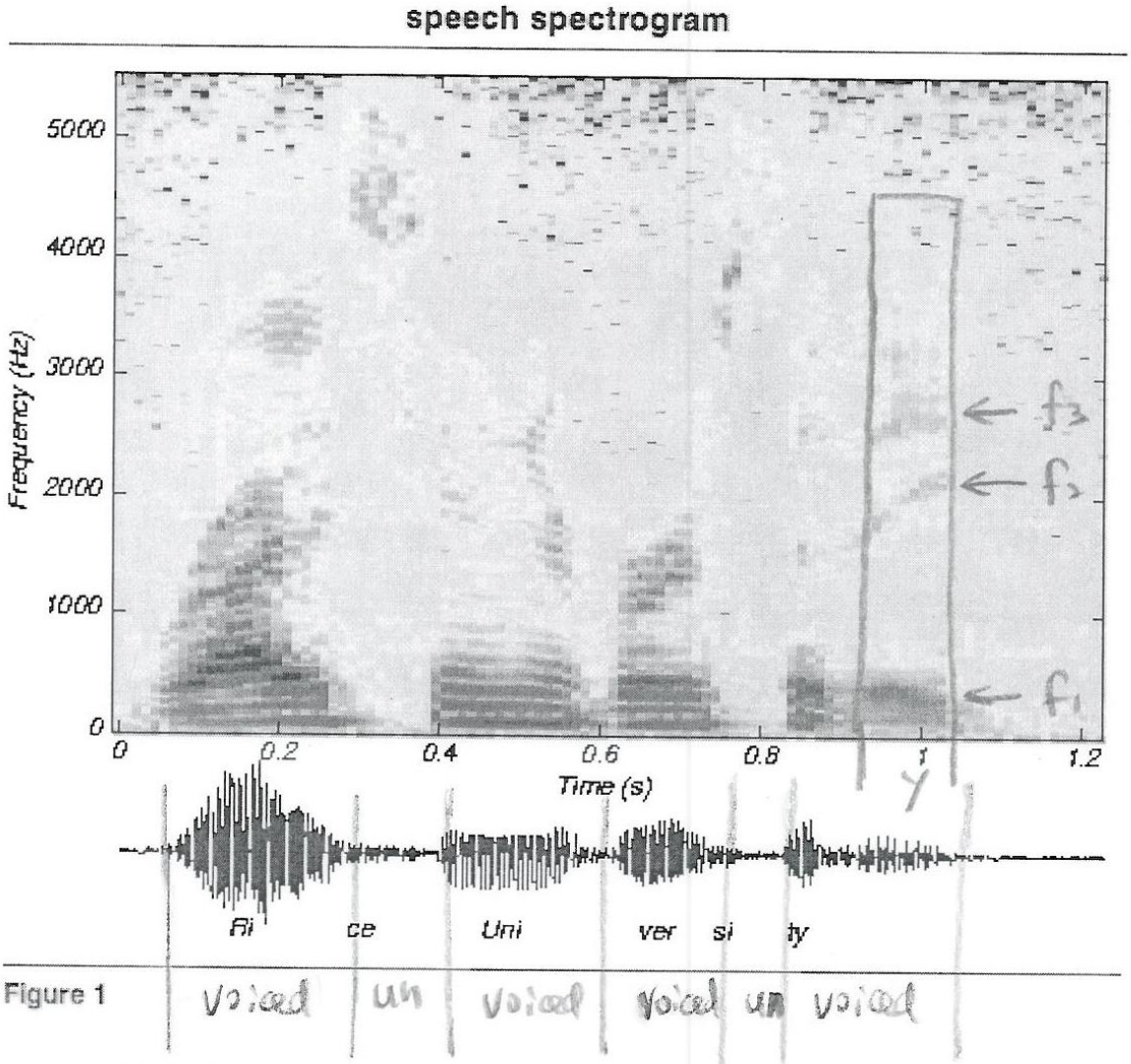


Figure 1

a. (5) Divide the 1.2 second duration of the utterance into voiced and unvoiced intervals.

b. (5)Determine the first three formant frequencies for the last part of the speech waveform corresponding to the "y" in "University". Your answer will only be approximate.

c. (5) Determine the approximate pitch frequency.

d. (5) Is the spectrogram shown wideband or narrowband?

e. (5) If your answer was "wideband", sketch what a narrowband spectrogram for this utterance would look like. If it was "narrowband", sketch what the wideband spectrogram would look like.

3. (25) Let us define the short-time continuous-time Fourier transform (STCTFT) $X(f,t)$ of the continuous-time signal $x(t)$ according to

$$X(f,t) = \int x(\tau) w(t-\tau) e^{-j2\pi f\tau} d\tau .$$

Here the function $w(t)$ denotes the window that is used to extract the short-time segment of the speech waveform. Suppose that

$$x(t) = \begin{cases} \cos(2\pi(500)t), & t \le 0 \\ \cos(2\pi(200)t), & t \ge 0 \end{cases}$$

and $w(t) = \text{rect}(100t)$

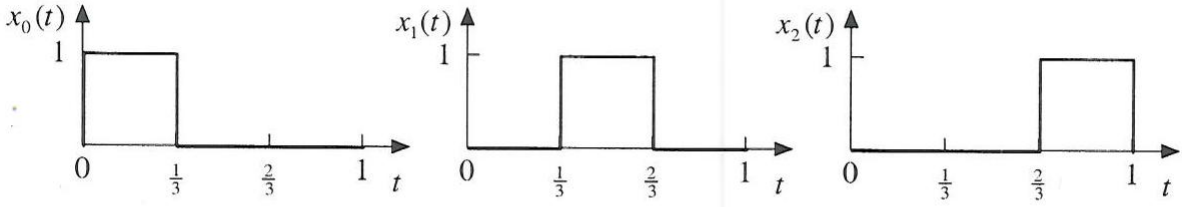
a. (7) Find a simple expression for the STCTFT at time $t \ge 0.005$. Sketch the magnitude of the STCTFT as function of frequency f for some fixed time t in this range.

b. (3) Find a simple expression for the STCTFT at time $t \le 0.005$. Sketch the magnitude of the STCTFT as function of frequency f for some fixed time t in this range.

c. (10) Find a simple expression for the STCTFT at time $t = 0$. Sketch the magnitude of the STCTFT as function of frequency f at time $t = 0$.

d. (5) Combining information from your answers to parts a. – c. above, sketch what a spectrogram plot of the STCTFT would look like for all t .

4. (25 pts) Consider the three functions shown below:



We wish to approximate the function $x(t) = t^2$ over the interval $0 \le t \le 1$ by the function $\hat{x}(t) = a_0 x_0(t) + a_1 x_1(t) + a_2 x_2(t)$.

a) (19) Find the coefficients a_0, a_1, a_2 that yield a minimum mean-squared error approximation.

b) (6) On the same axes, sketch $x(t)$ and $\hat{x}(t)$ for the range $0 \le t \le 1$.