- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (25 pts.) Let θ be a random variable that is uniformly distributed on the interval $(0, 2\pi)$.

   a. (7) Find the mean and variance of θ.

   Now, let $X = e^{j\theta}$ be a new random variable, where θ is defined as before.

   b. (6) Find $E\{X\}$ and $E\{|X|^2\}$, where $E\{\bullet\}$ denotes statistical expectation.

   Next, let $Y = A_1 e^{j\theta_1} + A_2 e^{j\theta_2}$ be a new random variable, where θ_1 and θ_2 are both uniformly distributed on the interval $(0, 2\pi)$, A_1 and A_2 are identically distributed with mean 0 and variance σ_A^2, and all four random variables θ_1, θ_2, A_1, and A_2 are mutually independent.

   c. (6) Find $E\{Y\}$ and $E\{|Y|^2\}$.

   Finally, let X_n, $n = \ldots., -2, -1, 0, 1, 2, \ldots$, be a wide-sense stationary random process with mean zero and autocorrelation $r_{xx}[n] = E\{X_m X_{m+n}\}$. Define a new random process $Y(\omega)$, $-\pi \leq \omega \leq \pi$, according to

$$Y(\omega) = \sum_{n=-N}^{N} X_n e^{-jwn}.$$

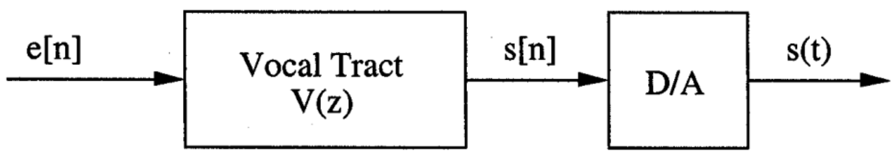
   d. (6) Find $E\{|Y(\omega)|^2\}$. Simplify your answer as much as possible, and discuss the significance of your result.

2. (25 pts.) A voiced speech waveform has pitch 200 Hz, and three formant frequencies at 0.5 kHz, 2 kHz, and 4 kHz, which decrease in amplitude with increasing frequency.

a. (7) Sketch a wideband spectrogram for this waveform. Be sure to label and dimension all important quantities.

b. (7) Sketch a narrowband spectrogram for this waveform. Be sure to label and dimension all important quantities.

We will use the digital system shown below to synthesize this speech waveform, where the excitation is given by $e[n] = \sum_{k=-\infty}^{\infty} \delta[n - Nk]$. Assume that the system operates at a 20 kHz sampling rate.



c. (11) Specify the value for the parameter N and the approximate location of all poles and zeros for the vocal tract response.

3. (25 pts.) The short-time discrete-time Fourier transform of the signal $x[k]$ is defined according to the equation

$$X(\omega,n)=\sum_k x[k]w[n-k]e^{-jwk},$$

Assume that we have a rectangular window

$$w[n]=\begin{cases}1, & |n|<10\\ 0, & \text{else}\end{cases}$$

and that our signal is given by

$$x[n]=\begin{cases}\cos(\pi n/3), & |n|<100\\ 0, & \text{else}\end{cases}$$

Find an exact expression for $X(\omega,n)$ for the following cases:

a. (8) $|n|<90$

b. (8) $|n|=100$

c. (1) $|n|>110$

d. (8) Based on your answers above, approximately sketch the complete spectrogram for all n.

4. (25 pts.) We observe the following 3 point signal $x[n]$:

n	0	1	2
$x[n]$	0	1	3

We wish to fit a straight line of the form $\hat{x}[n] = a_0 + a_1 n$ to these data points, where the coefficients a_0 and a_1 are chosen to minimize the mean-squared error

$$\varepsilon = \sum_{n=0}^{2} \left| \hat{x}[n] - x[n] \right|^2$$

a) (20) Find the optimal values for a_0 and a_1.

b) (5) Find the value of the mean-squared error ε for the choice of a_0 and a_1 given by your answer to part b).