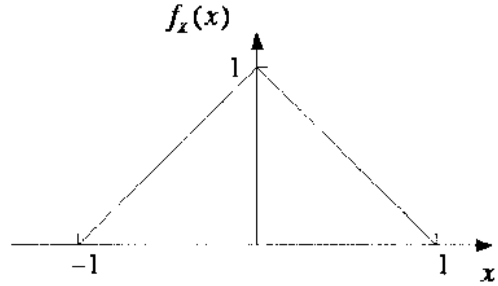
- You have 60 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (25 pts.) Consider a random variable X with probability density function $f_X(x)$ shown below.



a. (8) Find the mean μ_X and variance σ_X^2 for this random variable.

b. (6) Suppose that we quantize X to the two output levels −1/3 and 1/3. Find the mean square quantization error using the approximate expression derived in class.

c. (2) Find the signal-to-noise ratio due quantization based on your answers above.

d. (9) Find the exact mean-square quantization error.

2. (25 pts.)

Consider two random variables X and Y with mean zero, variance unity, and correlation coefficient ρ_{XY}. Let us define two new random variables U and V as follows:

$$U = X + Y$$

$$V = X - Y$$

(a) (8) Find the means and variances of U and V in terms of ρ_{XY}.

(b) (6) Find the correlation coefficient ρ_{UV} for U and V in terms of ρ_{XY}.

(c) (2) Show that it is possible to recover X and Y from U and V.

(d) (5) Suppose that we wish to quantize X and Y each at a fixed level of quantization noise power 10^{-4} Further, suppose we set the range of each quantizer to be 6 times the standard deviation of the random variable being quantized, i.e from -3σ to 3σ, where σ is the standard deivation of the random variable being quantized. How many bits will be required to quantize both X and Y?

(e) (3) Now assume that $|\rho_{XY}| = 0.9$, and suppose that instead, we quantize U and V, in the same manner as we quantized X and Y in part (d) above, i.e. we want the quantization noise power to be 10^{-4}; and we set the range of each quantizer to be 6 times the standard deviation of the random variable being quantized. How many bits will be required to quantize both U and V?

(f) (1) Discuss the significance of your results.

3. (25) A voiced speech waveform has pitch 100 Hz, and three formant frequencies at 1 kHz, 2 kHz, and 5 kHz, which decrease in amplitude with increasing frequency.

   a. (7) Sketch a wideband spectrogram for this waveform. Be sure to label and dimension all important quantities.

   b. (7) Sketch a narrowband spectrogram for this waveform. Be sure to label and dimension all important quantities.

   c. (11) Draw a block diagram of a digital system that could be used to synthesize this speech waveform. Assume that the system operates at a 20 kHz sampling rate. Be sure to define all components of the system and define all important parameters as accurately as possible.

4. (25 pts) Consider the signal $x(t) = (1 - t)^2,\ 0 \le t \le 1$. We want to approximate this signal over the interval $0 \le t \le 1$ by the signal $\tilde{x}(t) = a_0 + a_1 t$.

a) (12) Determine values for the coefficients a_0 and a_1 that will minimize the mean-square approximation error integrated over the range $0 \le t \le 1$.

b) (7) Carefully sketch the two functions $x(t)$ and $\tilde{x}(t)$ on the same axes.

c) (6) Compute the mean-square approximation error for the coefficient values that you determined in part a) above.