- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (30 pts.) Consider a causal linear, time-invariant system with transfer function

$$H(z) = \frac{1}{\left(1+z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}.$$

   a. (5) Find the region of convergence for $H(z)$. Is this system BIBO stable?

   b. (15) Find the impulse response $h[n]$.

   c. (10) Use the graphical approach to find the magnitude and phase of the frequency response at the frequency $\omega = \pi/2$ radians/sample.

2. (20 pts.)

a. (14) Find a simple expression for the N-point DFT $X[k]$ of the signal $x[n] = 2^{-n}, n = 0, \ldots, N-1$.

b. (6) Assuming that N is divisible by 8, use your results from part a to find a simple expression for the N-point DFT $X[k]$ of the signal.

$$x[n] = 2^{-n} \cos(\pi n / 4),\ n = 0, \ldots, N-1.$$

3. (20 pts.) The signal $x(t) = 2\cos(2\pi(800)t) + \cos(2\pi(5000)t)$ is sampled at $N = 32$ points; and the 32-point DFT $X^{(32)}[k],\ k = 0,\ldots,31$ is computed.

a. (12) Find the approximate values for k where we would expect to observe peaks in the DFT. What would be the approximate magnitude of the DFT at each of these k-values?

b. (8) Very roughly sketch the 32-point DFT indicating where aliasing and/or truncation effects and leakage are observed.

4. (30 pts.)

a) (20) Derive the equations for the $N = 12$ point Fast Fourier Transform algorithm.

b) (10) Determine the computation required to compute your 12 point FFT. Compare with the computation for direct evaluation of a 12 point discrete Fourier transform.