- You have 60 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (25 pts.) Consider the causal LTI system described by the following difference equation

$$y[n] = x[n] + \tfrac{1}{2} y[n-1]$$

Suppose that the signal $x[n] = 2^n u[-n]$ is input to this system.

a. (12) Find the Z-transform $Y(z)$ of the output $y[n]$. Be sure to state the region of convergence for $Y(z)$.

b. (2) Is the system stable? State why or why not.

c. (11) Find the system output $y[n]$ by finding the inverse Z-transform of $Y(z)$.

2. (25 pts.) Consider an N-point signal $x[n], n = 0, \ldots, N-1$, where N is even. Suppose we generate a new N/2-point signal $y[n]$ by taking every other data point from $x[n]$, i.e. $y[n] = x[2n], n = 0, \ldots, \frac{N}{2} - 1$.

Find a simple expression for the N/2-point DFT $Y^{(N/2)}[k]$ of $y[n]$ in terms of the N-point DFT $X^{(N)}[k]$ of $x[n]$.

3. (25) The signal $x(t) = \cos(2\pi(110)t)$ is sampled 16 times at a 320 Hz rate to obtain $x[n], n = 0, \ldots, 15$. We then compute the 16-point DFT $X^{(16)}[k]$ of this signal.

a. (7) Determine the values of k corresponding to the peaks that we would observe in the DFT $X^{(16)}[k]$.

b. (2) Are picket fence and leakage present in this case?

c. (10) Find an expression for the DFT $X^{(16)}[k]$. You may use anything from the formula sheet "ECE 438 Essential Definitions and Relations" to solve this problem.

d. (6) Sketch the DFT $X^{(16)}[k]$.

4. (25 pts) You have a subroutine for the N-point radix-2 FFT where N is any power of 2. You wish to compute an exact 24-point DFT.

a) (10) Derive a set of equations that shows how the 24-point DFT can be efficiently calculated by using your radix-2 FFT subroutine.

b) (9) Draw a block diagram for your 24-point FFT algorithm. Do not show any internal details for the radix-2 part of this algorithm. It should be treated as a black box.

c) (2) Find the approximate number of complex operations (each complex operation consists of one complex multiplication and one complex addition) required to directly compute the 24-point DFT.

d) (4) Find the approximate number of complex operations required to compute the 24-point DFT using your 24-point FFT algorithm.