EE 438 Solution **Exam No. 2** **Spring 2003**

- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- Unless explicitly stated to the contrary, you must simplify your answer as much as possible to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (25 pts.) Consider a causal linear time-invariant system with input $x[n] = \left(\frac{1}{2}\right)^n u[n]$ and output $y[n] = \left[2 - \left(\frac{1}{2}\right)^n\right]u[n]$.

   a) (13) Find the Z transform $Y(z)$. Be sure to state the region of convergence.

   b) (5) Find the transfer function $H(z)$ for the system.

   c) (2) Is the system bounded-input-bounded-output stable? Explain why or why not.

   d) (5) Find the impulse response $h[n]$ for this system.

2. (25 pts.) Consider the following 20 point signal

$$x[n] = \begin{cases} 1, & n = 0,1,\dots,9 \\ 0, & n = 10,11,\dots,19 \end{cases}$$

a) (19) Find a simple expression for the 20-point DFT $X[k]$ of this signal.

b) (6) Carefully sketch $X[k]$.

3. (25 pts.) You have an integrated circuit chip that computes the radix 2 FFT for length 64 points. You would like to use one or more of these chips to efficiently compute a 192 point DFT by embedding the 64-point FFT chips in an application specific integrated circuit (ASIC).

a) (10) Derive an expression for the 192 point DFT in terms of the 64-point DFT.

b) (10) Draw a flow diagram that shows the layout of the ASIC. Be sure to completely label your diagram showing all twiddle factors.

c) (2) How many 64-point FFT chips will be required?

d) (3) Compute the required computation to calculate the 192-point DFT in terms of the number of complex operations.

4. (25 pts) Consider the following signals

n	0	1	2	3	4
$x[n]$	3	2	0	–3	–2
$y[n]$	1	1	1	0	0

a) (10) Calculate the aperiodic convolution of $x[n]$ and $y[n]$.

b) (15) Calculate the periodic (circular) convolution of $x[n]$ and $y[n]$ with period 5.

c) (3) To what length must these signals be padded with zeros so that a portion of the periodic convolution will yield the same result as the non-zero part of the aperiodic convolution?