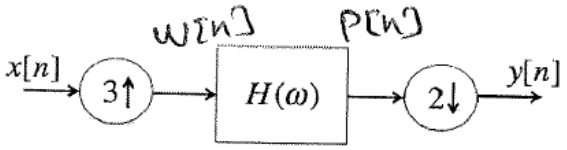
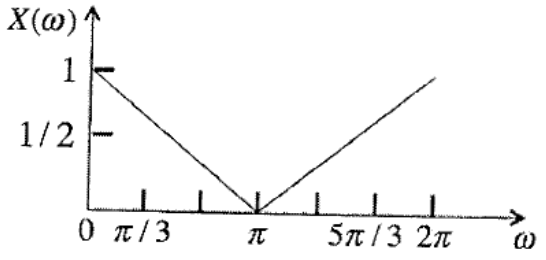
- You have 50 minutes to work the following four problems.
- Be sure to show all your work to obtain full credit.
- You do *not* need to derive any result that can be found on the formula sheet. However, you should state that it can be found there.
- The exam is closed book and closed notes.
- Calculators are permitted.

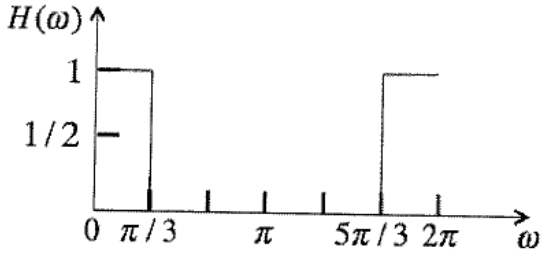
1. (25 pts.) Consider the multi-rate signal processing system shown below



with input $x[n]$ having DTFT $X(\omega)$. The filter between the upsampler and the downsampler has frequency response $H(\omega)$.

a. (15) Find a simple and complete, general expression for the DTFT $Y(\omega)$ of the output $y[n]$ in terms of the DTFT $X(\omega)$ of the input and the frequency response $H(\omega)$ of the filter.





b. (8) For the specific input DTFT $X(\omega)$ shown above and the specific filter frequency response $H(\omega)$ shown above, plot the DTFT $Y(\omega)$ of the output.

c. (2) Based on your answer to part (b), what is the overall effect of this system on the sampling rate?

2. (25 pts.) Consider a discrete-time, linear, time-invariant system described by the following difference equation:

$$y[n] = x[n] + \frac{1}{3} y[n-1]$$

Suppose that the input to this system is $x[n] = \left(\frac{1}{2}\right)^n u[n]$. Use Z transform techniques to find a simple and complete expression for the output $y[n]$.

3. (25) Consider the 8-point signal

$$x[n] = \begin{cases} \cos(5\pi n/8), & n = 0,\ldots,7 \\ 0, & \text{else} \end{cases}$$

a) (10) Find a simple, closed form expression for the DTFT $X(\omega)$ of this signal.

b) (5) Sketch $X(\omega)$ being sure to label the zero crossings along the frequency axis.

c) (5) Find a general expression for the 8-point DFT $X[k]$ of $x[n]$ in terms of its DTFT $X(\omega)$.

d) (5) Combine your answers to parts (b) and (c) to sketch the DFT $X[k]$ of $x[n]$.

4. (25 pts)

a) (17) Derive the complete set of equations for the exact 15-point decimation-in-time Fast Fourier Transform (FFT) algorithm. (You do *not* need to provide a flow diagram for the algorithm.)

b) (3) Determine the number of complex operations required to implement the 15-point Discrete Fourier Transform (DFT) directly. (A complex operation is defined to consist of one complex multiply *and* one complex addition.)

c) (5) Determine the number of complex operations required to implement the 15-point Discrete Fourier Transform (DFT) using a 15-point FFT algorithm. (A complex operation is defined to consist of one complex multiply *and* one complex addition.)