ECE 438 lecture Friday 31 March 2023

Announcements:

To be posted at the course website:

1. Solution to Exam #2
2. HW #8
3. Solution to HW #7
4. Today's recording and Notability PDF

Continue discussion of STDTFT

Review

def. $\tilde{S}(n_0, \omega) = \sum_n s[n]\, w[n_0 - n]\, e^{-j\omega n}$

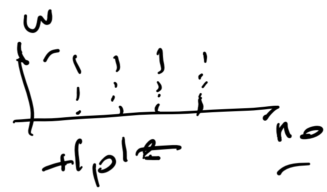
$w[-(n-n_0)]$

length of $w[n]$ – N

pitch period in samples P

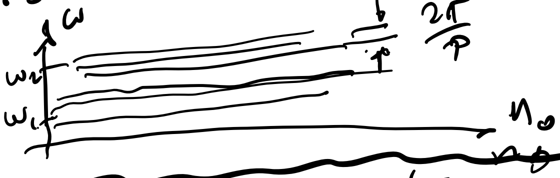
Spectrograms

1. wideband $N \approx P$ or $N < P$



2. narrowband $N \gg P$

for one phoneme



Use commutativity of convolution

$$\tilde{S}(n_0, \omega) = \sum_n s[n_0 - n]\, w[n]\, e^{-j\omega(n_0 - n)}$$

Have not changed anything!

$$\tilde S(n_0,\omega) = e^{-j\omega n_0} \sum_n s[n_0-n] \underbrace{w[n] e^{j\omega n}}_{h_\omega[n]}$$

downshift

$h_\omega[n]$ filter unit sample response

upshift

In frequency domain

now treat n_0 as the time variable, call it n

then we have $\tilde S(n,\omega_0) = e^{-j\omega_0 n}\left[s[n] * h_{\omega_0}[n]\right]$

now look at frequency domain:

(let $\omega = \omega_0$ fixed)

$$\mathcal{S}(\omega,\omega_0) = \sum_n \tilde S[n,\omega_0]\, e^{-j\omega n}$$

DTFT of $\tilde S[n,\omega_0]$ with respect to n

$$= S(\omega')\, H_{\omega_0}(\omega')\Big|_{\omega' = \omega + \omega_0}$$

have a NB filter with frequency response $H_{\omega_0}(\omega)$ centered at ω_0

Next: discretize frequency!

$$\omega = \omega_r = 2\pi r$$

N

now we are switching N from length of $W[n]$ to number of samples in frequency domain:

$$\tilde{S}(\omega_0, n) \Rightarrow S\left(\frac{2\pi r}{N}, n\right) \triangleq S_r[n]$$

$$S_r[n] = \sum_k s[k]\, h[n-k]\, e^{-j\frac{2\pi k r}{N}}$$

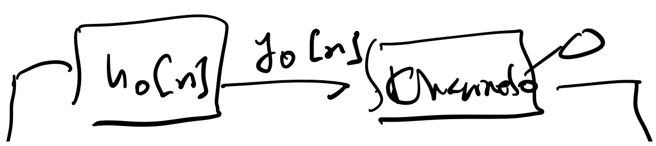
$$= \sum_k s[n-k]\, h[k]\, e^{-j\frac{2\pi (n-k) r}{N}}$$

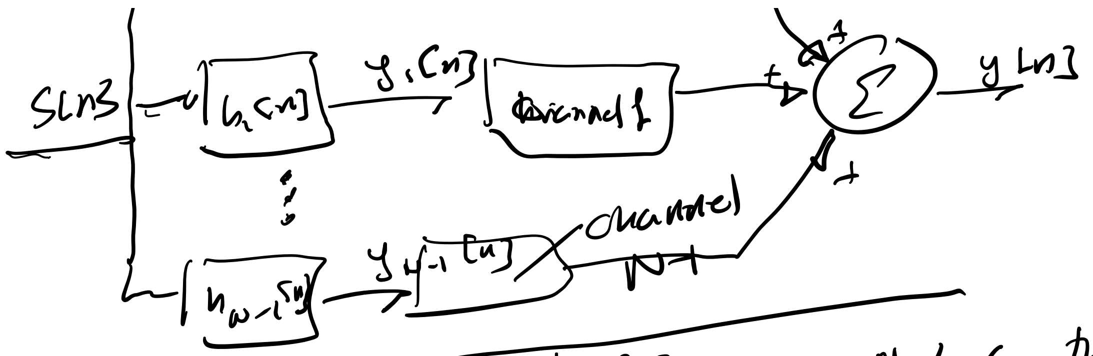
$$= \underbrace{e^{-j\frac{2\pi r n}{N}}}_{\text{downshift}} \sum_k s[n-k]\, \underbrace{h[k]\, \overset{\text{upshift}}{e^{+j\frac{2\pi k r}{N}}}}_{h_r[n]}$$

define $y_r[n] = e^{j\frac{2\pi r n}{N}} S_r[n]$ to cancel the downshift

$$= \sum_k s[n-k]\, h_r[k], \quad r = 0, \ldots, N-1$$

What we have:





Goal: Design $h_r[n]$, $r=0,\dots,N-1$ so that

$$y[n] \equiv s[n]$$

Why?

Each channel can be allocated a number of bits (bandwidth) that reflects the perceptual significance of that channel

i.e. source coding

$$H_r(\omega) = H_0\left(\omega - \frac{2\pi r}{N}\right) = W\left[\omega - \frac{2\pi r}{N}\right]$$

DTFT of window $w[n]$

this is a modulated filter bank

This is only one type of filter bank (see HW #8)

Goal: $y[n] \equiv s[n] \Rightarrow$ perfect reconstruction (PR)

what requirement does $h_0[n] = w[n]$ need to satisfy for PR

On HW #8, you will show that when $h_0[n]$ is an ideal low pass filter, we have PR

The problem is that an ideal lowpass filter has a unit sample response with infinite duration $\Rightarrow$ it is not realizable

recall that

$$y[n] = \sum_{r=0}^{N-1} y_r[n]$$

$$= \sum_{r=0}^{N-1} s[n] * h_r[n]$$

In frequency domain (DTFT), we have

$$Y(\omega) = \sum_{r=0}^{N-1} S(\omega) H_r(\omega)$$

$$= \sum_{r=0}^{N-1} H_r(\omega) S(\omega)$$

for $Y(\omega) = S(\omega)$ (PR), must have

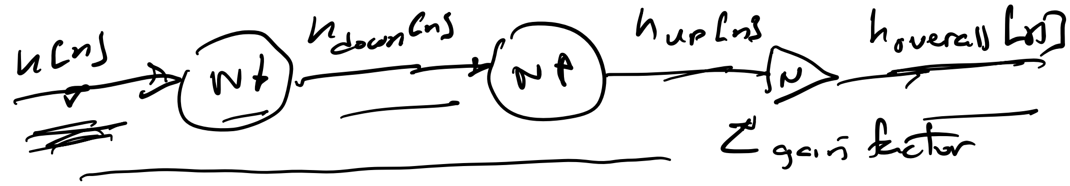
$$\sum_{r=0}^{N-1} H_r(\omega) \equiv 1 \qquad (1)$$

What does this tell us about $h[n]$?

Look at margin notes of Module 4.2.4

watch for error — depend on recording made now.

Consider the following system:



want $H_{overall}(\omega) \equiv 1 \Rightarrow h_{overall}[n] = \delta[n]$

Now look at system in frequency domain

$$H_{down}(\omega) = \frac{1}{N} \sum_{r=0}^{N-1} H\left(\frac{\omega - 2\pi r}{N}\right)$$

also

$$H_{up}(\omega) = H_{down}(\omega N)$$

$$= \frac{1}{N} \sum_{r=0}^{N-1} H\left(\frac{N\omega - 2\pi r}{N}\right)$$

$$= \frac{1}{N} \sum_{r=0}^{N-1} H\left(\omega - \frac{2\pi r}{N}\right)$$

$H_{overall}(\omega) = N H_{up}(\omega)$ (2)

now we want $H_{overall}(\omega) \equiv 1$

but $h_{overall}[n] = N h_{up}[n]$

$\Rightarrow$ (2)

$$\Rightarrow \underbrace{\sum_{r=0}^{N-1} H\left(\omega - \frac{2\pi r}{N}\right)}_{H_r(\omega)} \equiv 1 \quad \text{Same as (1)}$$