ECE 438 lecture 29 March 2023 (Wednesday)

Announcements

- HW #7 due today at 11:59p EDT on Gradescope
- Office Hours
  - 3:30p EDT – Allebach
  - 12:30p EDT – Hossain

STDTFT

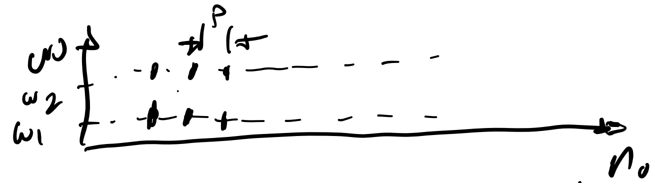
$$\tilde{S}(\omega, n_0) = \sum_{n=-\infty}^{\infty} s[n] \underbrace{w[n_0 - n]}_{w[-(n-n_0)]} e^{-j\omega n}$$

1ST interpretation

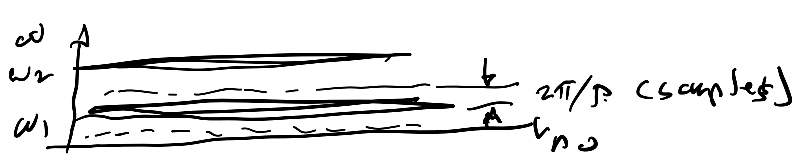
window duration is N (samples)

pitch period P (samples)

① Wideband spectrogram ($N \approx P$ or $N < P$)



② narrowband spectrogram ($N >> P$)



1st interpretation: DTFT of a windowed segment of $s[n]$ where the window is centered at n_0

$$\tilde{S}(\omega, n_0) = \sum_n s[n]\, w[n_0 - n]\, e^{-j\omega n}$$ ← def. of STDTFT (Eq. (1))

Commutativity of convolution of $y[n]$ & $x[n]$:

$$\sum_k y[k]\, x[n-k] = \sum_k y[n-k]\, x[k]$$

$y[-(k-n)]$

apply this to STDTFT (Eq. (1))

$$\tilde{S}(n, n_0) = \sum_n w[n]\, s[n_0 - n]\, e^{-j\omega(n_0 - n)}$$

$$= e^{-j\omega n_0} \sum_n w[n]\, e^{+j\omega n}\, s[n_0 - n]$$

Block diagram:

Recall $e^{j\omega_0 n} x[n] \xrightarrow{DTFT} X(\omega - \omega_0)$

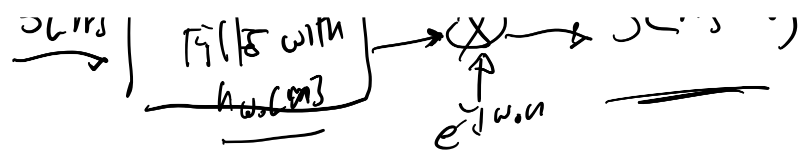
Fix $\omega = \omega_0$, replace n_0 by n

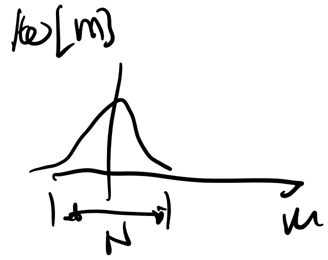
then we have

STDTFT $\sim \tilde{S}(n, \omega_0) = e^{-j\omega_0 n} \sum_m \underbrace{w[m]\, e^{j\omega_0 m}}_{h_{\omega_0}[m]}\, s[n-m]$

Finally, we get to the block diagram

$s[n]$ $\tilde{S}(n, \omega_0)$





What is happening frequency domain?

An Answer is: I thought we were in the frequency domain!!!

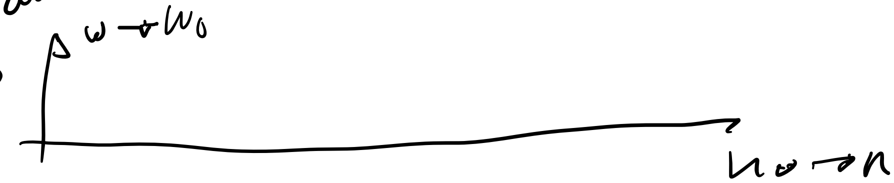
But we changed the time variable from n to n_0 – location of center of window

Now let's define a new DTFT:

$$S(\omega, \omega_0) = \sum_n \tilde{s}(n, \omega_0) e^{-j\omega n} \quad \text{Eq.(2)}$$

capital S

Spectrogram view

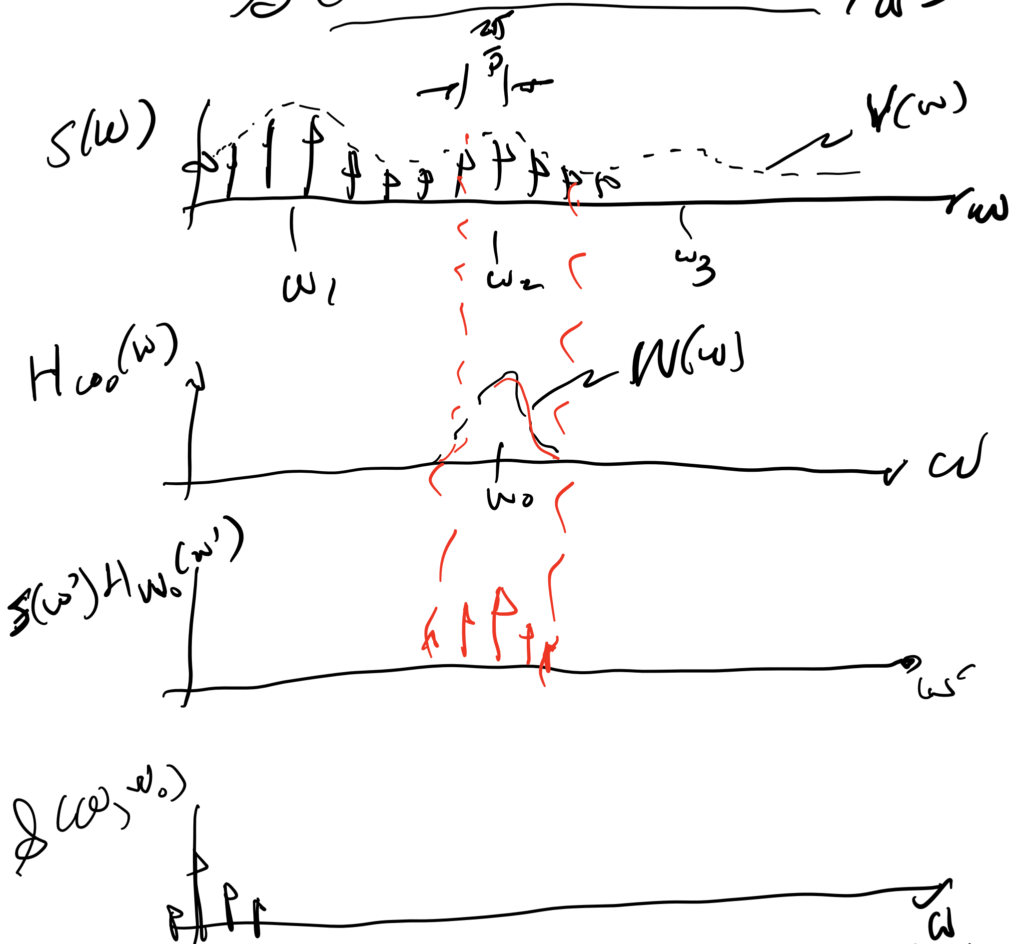


Take the DTFT with respect to the center of the window

recall that $\hat{S}(n, \omega_0) = e^{-j\omega_0 n}\left[s[n] * h_{\omega_0}[n]\right]$

Using Eq. (2), we then have

$$\mathcal{S}(\omega, \omega_0) = \underline{S(\omega') H_{\omega_0}(\omega')}\Big|_{\omega = \omega' - \omega_0}$$



This is the second interpretation of the STDTFT:
i.e. DTFT of NB-filtered s[n] shifted down to baseband where the NB filter is centered at n_0

See Module 4.2.2 (formally prepared)

Recap: steps to generate STDTFT (actually $\mathcal{S}(\omega, \omega_0)$):

Scheme:

1. Take DTFT of $w(n)$
2. Upshift this to ω_0; this is $H_{\omega_0}(\omega)$
3. Multiply by $S(\omega)$
4. Downshift product by ω_0 to center the spectrum at DC ($\omega=0$)

Filterbanks

sample frequency axis at N pts.

$$\omega_k = \frac{2\pi k}{N}, \quad k=0,\ldots,N-1$$

abuse of notation: N is no longer the length of the window, it is the sampling interval in the frequency domain. We don't need to talk about the length of the window any more.

This material is in Module 4.2.4 (handwritten)

The fact that we sampling the frequency domain at N uniformly spaced points looks like the DFT. But, we are not summing over a finite range $n=0,\ldots,N-1$

It is possible to express these results in

It is possible to [illegible]
terms of the N-pt. DFT & therefore use the FFT for efficient evaluation of the STDTFT. But we will not talk about this in ECE 438.

$$\tilde{S}(\omega_0, u) \Rightarrow S\left(\frac{2\pi r}{N}, u\right) \triangleq S_r[n]$$

rewriting definition of STDTFT in ~~terms~~ of $S_r[n]$, we have

$$S_r[n] = \sum_k s[k]\, w[n-k]\, e^{-j\frac{2\pi k r}{N}}$$