ECE 438 Lecture Monday March 20, 2023

Announcements

- Office Hours today

  2:30p - 3:20p EDT

  4:00p - 5:00p EDT

- Exam 2 will be given during regular class period Wednesday March 22, 2023

  Please bring photo ID

Random Sequences (Random DT signals)

Expectation

Module 3.1.5

Filtering of random signals

define $\bar{X} = \mathcal{E}\{X_n\}$ consider it a constant

$$r_{XX}[m,n] = \mathcal{E}\{\underline{X_n X_m}\}$$

Special case:

A process is wide-sense stationary (w.s.s.)

if ① $\bar{X}_n$ is constant, ie $\mathcal{E}\{X_n\} = \bar{X}$ does not depend on n

② $r_{XX}[m,n] = r_{XX}[n-m, 0]$

<u>Convention</u>: $r_{AB}[a,b] = r_{AB}[b-a]$

doesn't apply to autocorrelation because

$r_{xx}[n-m] = r_{xx}[m-n]$ by definition

XX

<u>Now consider filtering</u>:

$$y_n = \sum_{m=-\infty}^{\infty} h[n-m]\, x_m$$

$$\mathcal{E}\{y_n\} = \sum_{m=-\infty}^{\infty} h[n-m]\, \mathcal{E}\{x_n\} = \sum_{m=-\infty}^{\infty} h[n-m]\, \bar{x}$$

assuming $\bar{x}_n$ doesn't depend on n

$$= \left(\sum_{m=-\infty}^{\infty} h[m]\right) \bar{x}$$

Suppose that x_n is w.s.s.

Want to find $r_{yy}[m,n] = \mathcal{E}\{y_m\, y_n\}$

Start with cross-correlation:

$$r_{xy}[m,n] = \mathcal{E}\{x_m\, y_n\}$$

$$= \mathcal{E}\{x_m \sum_{k=-\infty}^{\infty} h[n-k]\, x_k\}$$

$$= \sum^{\infty} h[n-k]\, \mathcal{E}\{x_m\, x_k\}$$

$$= \sum_{k=-\infty}$$

$$= \sum_{k=-\infty}^{\infty} h[n-k]\, r_{XX}[k-m]$$

let $l = k-m \Rightarrow k = l+m$

$$r_{XY}[m,n] = \sum_{l=-\infty}^{\infty} h[n-(l+m)]\, r_{XX}[l]$$

look for error in posted notes

$$= \sum_{l=-\infty}^{\infty} h[n-m-l]\, r_{XX}[l]$$

Convolution for $r_{XX}[l]$ with the filter impulse response $h[l]$

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now let's consider

$$r_{YY}[m,n] = E\{Y_m Y_n\}$$

$$= E\{Y_m \sum_{k=-\infty}^{\infty} h[n-k] X_k\}$$

$$= \sum_{k=-\infty}^{\infty} h[n-k]\, E\{Y_m X_k\}$$

$$= \sum_{k=-\infty}^{\infty} h[n-k]\, r_{XY}[m-k]$$

let $l = m-k \Rightarrow k = m-l$

$$r_{yy}[m,n] = \sum_{l=-\infty}^{\infty} h[n-(m-l)]\, r_{xy}[l]$$

$$= \sum_{l=-\infty}^{\infty} h[n-m+l]\, r_{xy}[l]$$

need this to be a - for it to look like convolution

let $l' = -l$

$$r_{yy}[m,n] = \sum_{l'=-\infty}^{\infty} h[n-m-l']\, r_{xy}[-l']$$

$$= r_{yy}[m-n]$$

So if the input X_n to the filter is w.s.s., then the output is also w.s.s.!!

block diagram to illustrate the concept: