# ECE 438 lecture Wednesday 1 March 2023

## Announcements

1. Office Hour today at 3:30p EST
2. HW #5 due tonight at 11:59p EST

we have $x[m] \neq 0$ only for $0 \leq m \leq N-1$

filter $h[m] \neq 0$, only for $0 \leq m \leq M-1$

assume $M < N$

want to evaluate $y[n] = \sum_{m=0}^{N-1} x[m]\, h[-(m-n)]$

$C_{DIRECT} = MN$ c.o. $= 654{,}850$ FLOPS

$C_{FFT} = 3N\log_2(N) + \frac{N}{2}$ c.o. $= 134{,}502$ FLOPS

$N = 1024$

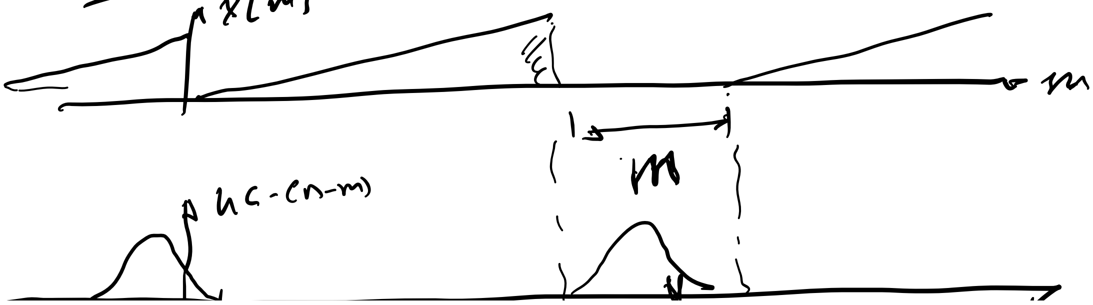
$$\eta = \frac{C_{DIRECT}}{C_{FFT}} = \frac{M}{3\log_2(N) + \frac{1}{2}} \gg 1$$

$\eta = \underline{4.87}$

$\underline{L = 2048}$:  $C_{FFT} = 289{,}165$

$\eta = \underline{2.26}$

## illustration



$n$ $\qquad$ $n+(N+M-1)$ $\qquad$ $M$

$|\xleftrightarrow{M}|$

This concludes discussion of efficient filtering using the FFT

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under Foundations — posted lecture notes

Module 1.7 Digital filtering

Module 1.8 FM Stereo

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Module 3 Random signals

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Module 3.1.1.1 One random variable (r.v.)

The posted notes are handwritten

A r.v. is completely characterized by its density function $f_X(x)$

Note: we use upper case letters to denote r.v.'s

This is an "abuse of notation"

$$P\{a < X \le b\} = \int_a^b f_X(x)\,dx$$

$\Rightarrow$ $f_X(x)\Delta x \cong P\{x \le X \le x+\Delta x\}$ for small $\Delta x$

Also define distribution function

$$F_X(x) = P\{X \le x\} = \int_{-\infty}^{x} f_X(\xi)\,d\xi$$

$$\Rightarrow \quad \frac{dF_X(x)}{dx} = f_X(x)$$

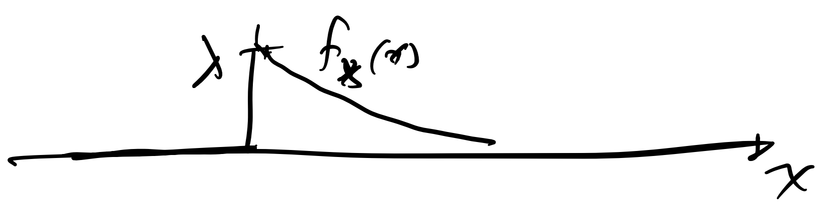
Properties of density functions:

① $f_X(x) \geq 0$

② $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

Example:

$$f_X(x) = \lambda e^{-\lambda x}\, u(x) \qquad \lambda = \text{constant} > 0$$




$f_X(x) \geq 0$ by inspection

consider $\int_{-\infty}^{\infty} f_X(x)\,dx = \int_0^{\infty} \lambda e^{-\lambda x}\,dx$

$$= \Big|_0^{\infty} \lambda\left(\frac{1}{-\lambda}\right) e^{-\lambda x}$$

$$= \Big|_{+\infty}^{0} e^{-\lambda x} = 1 - 0 = 1 \quad ✓$$

$$P\{X \le 1\} = \int_0^1 \lambda e^{-\lambda x} dx = -e^{-\lambda x}\Big|_1^0 = 1 - e^{-\lambda}$$

## Expectation

$$\mathcal{E}\{g(X)\} = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$ where $g$ is a function

↑
script
$E$

Special cases:

① mean $\mathcal{E}\{X\} = \bar{X} \Rightarrow g(x) = x$

so $\bar{X} = \int_{-\infty}^{\infty} x f_X(x) dx$

② second moment

$\mathcal{E}\{X^2\} \Rightarrow g(x) = x^2$

Notation: $\mathcal{E}\{X^2\} = \overline{X^2}$ don't confuse this with $(\bar{X})^2$

$$\overline{X^2} = \int_{-\infty}^{\infty} x^2 f_X(x) dx$$

③ Variance $g(x) = (x - \bar{X})^2$

$$\sigma_X^2 = \mathcal{E}\{(X-\bar{X})^2\}$$

property

expectation is linear

$$\mathcal{E}\{a\,g(X) + b\,h(X)\} = a\,\mathcal{E}\{g(X)\} + b\,\mathcal{E}\{h(X)\}$$

example

$$\sigma_X^2 = \mathcal{E}\{(X^2 - 2X\bar{X} + (\bar{X})^2)\}$$

$$= \underbrace{\mathcal{E}\{X^2\}}_{\overline{X^2}} - 2\bar{X}\,\mathcal{E}\{X\} + (\bar{X})^2 = \overline{X^2} - (\bar{X})^2$$