ECE 438 Lecture 1/30/2023

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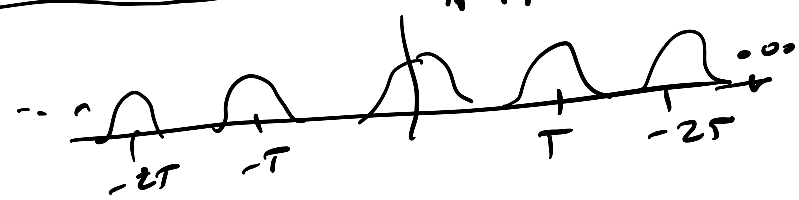
Announcements

1. Office Hours today at 4p EST
2. Quiz No. 1 available at 6p EST on Gradescope

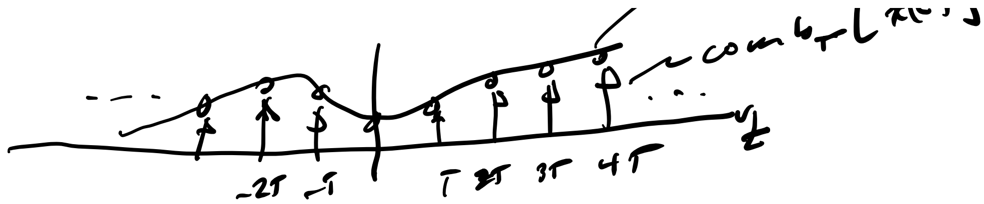
Two operators:

1. $$\mathrm{rep}_T[x(t)] = \sum_{k=-\infty}^{\infty} x(t-kT)$$



2. $$\mathrm{comb}_T[x(t)] = \sum_{k=-\infty}^{\infty} x(kT)\,\delta(t-kT)$$

x(t)



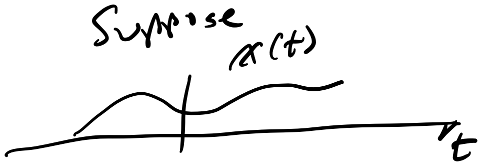
(a) $\text{rep}_T[x(t)] \xleftrightarrow{\text{CTFT}} \frac{1}{T}\,\text{comb}_{\frac{1}{T}}[X(f)]$

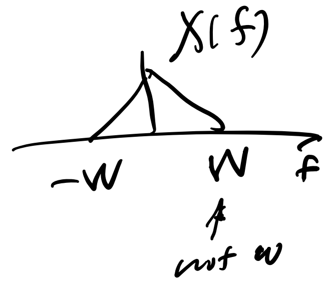
where $x(t) \xleftrightarrow{\text{CTFT}} X(f)$

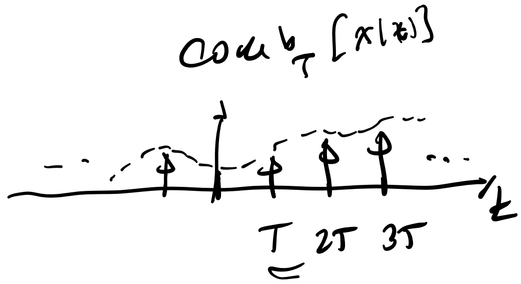
(b) $\text{comb}_T[x(t)] \xleftrightarrow{\text{CTFT}} \frac{1}{T}\,\text{rep}_{\frac{1}{T}}[X(f)]$

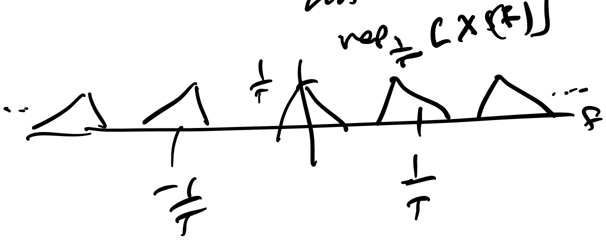
Example

Suppose



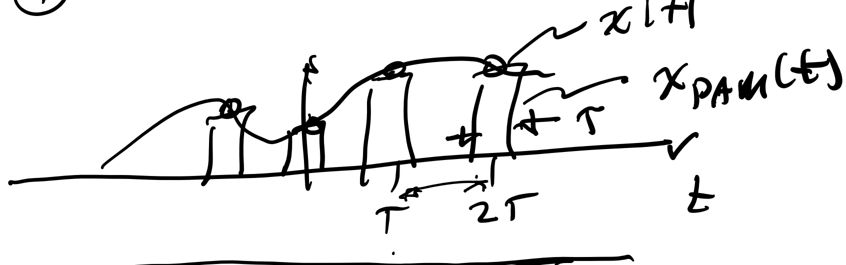






More practical example

(1) Pulse amplitude modulation (PAM)



find $X_{PAM}(f)$ in terms of $X(f)$

Task: find $x_{PAM}(f)$ in terms of $X(f)$

Solution:

Let $x_{PAM}(t) = \underbrace{comb_T[x(t)] * rect\left(\frac{t}{\tau}\right)}$

$$= \left[\sum_{k=-\infty}^{\infty} x(kT)\,\delta(t-kT)\right] * rect\left(\frac{t}{\tau}\right)$$

interchange

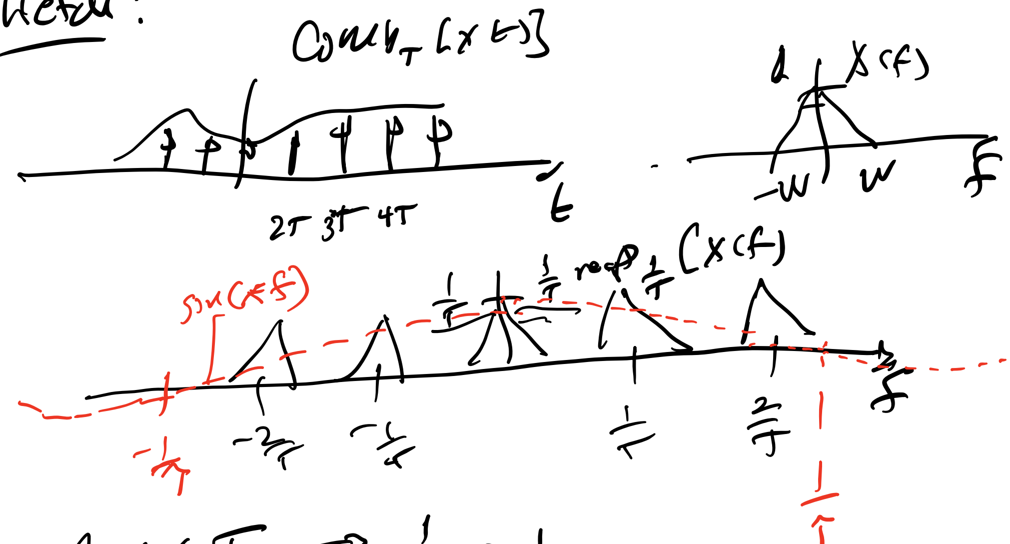
$$x_{PAM}(t) = \sum_{k=-\infty}^{\infty} x(kT)\left[\delta(t-kT) * rect(t/\tau)\right]$$

Property: $y(t) * \delta(t-t_0) = y(t-t_0)$

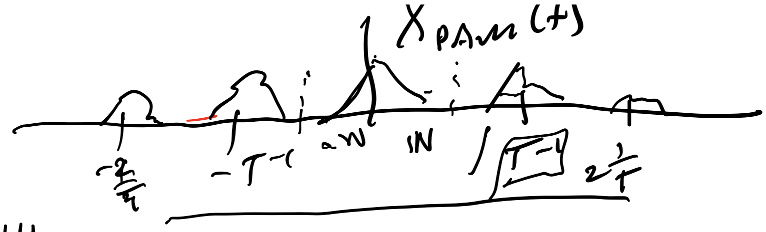
$$\therefore\ x_{PAM}(t) = \sum_{k=-\infty}^{\infty} x(kT)\ rect\left(\frac{t-kT}{\tau}\right)$$

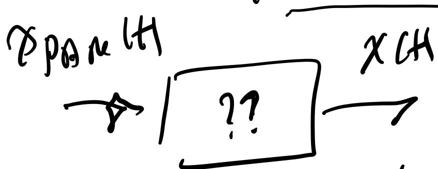
$$X_{PAM}(f) = \frac{1}{T} rep_{\frac{1}{T}}[X(f)] \cdot \tau\, sinc(\tau f)$$

Sketch:

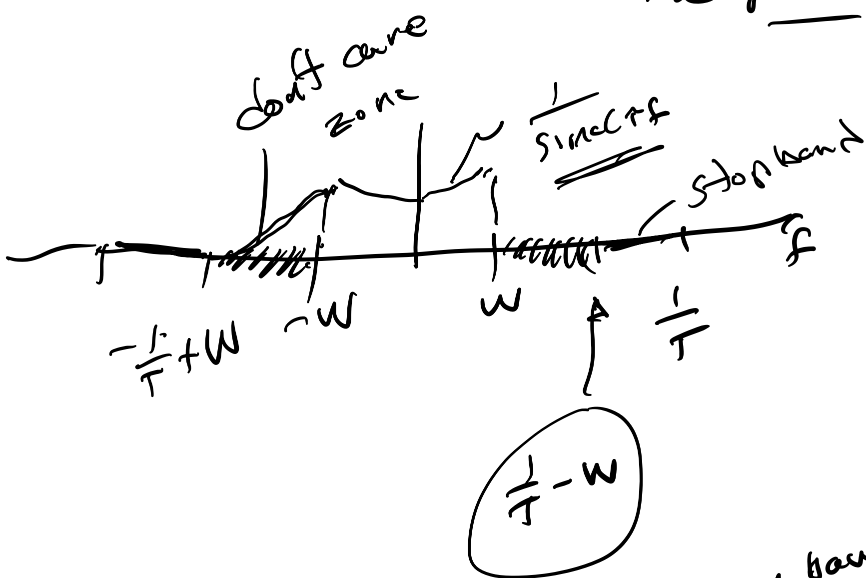


$\tau << T \Rightarrow \frac{1}{\tau} >> \frac{1}{T}$

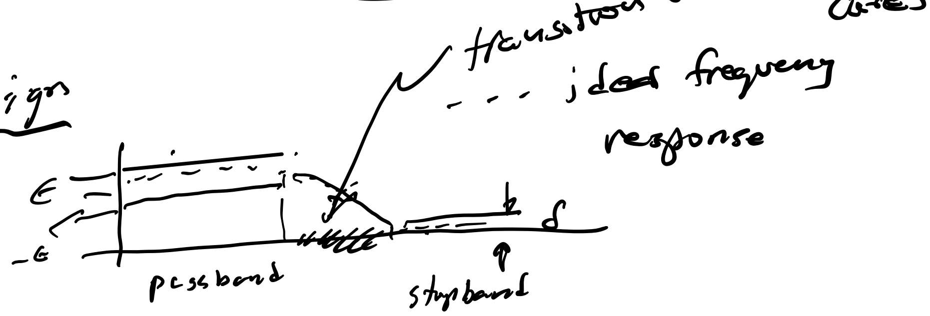




What goes here? (A low pass filter with amplification of high frequencies in the passband



Filter design



Example 2 Sampling a cosine

$$x(t) = \cos(2\pi(3000)t)$$

$$f_s = \frac{1}{T} = 5000\ \text{Hz} = 5\text{kHz}$$

$\llcorner$ sampling frequency

"Ideal sampler" $\Rightarrow$ use comb operator

$$\underbrace{x_s}_{\text{sampled}}(t) = \text{comb}_T[x(t)]$$

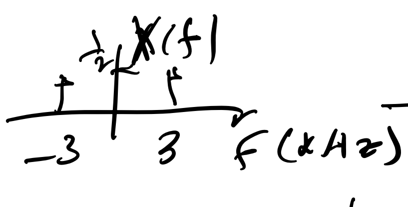
$$X_s(f) = \frac{1}{T}\,\text{rep}_{\frac{1}{T}}[X(f)] = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(f - k5000)$$

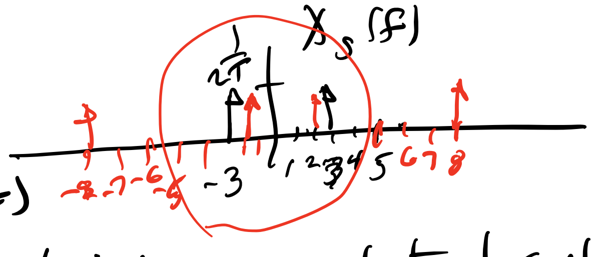
$$X(f) = \frac{1}{2}\{\delta(f-3000) + \delta(f+3000)\}$$

CTFT is not periodic in frequency $\Rightarrow$ don't need domain restrictions like

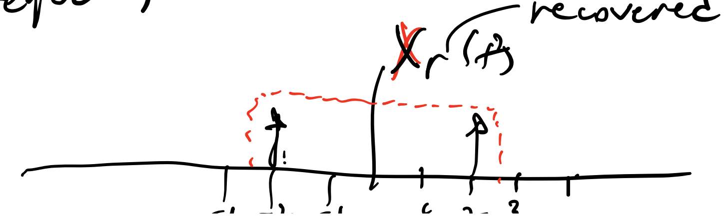
$0 \le \omega \le 2\pi$ or $-\pi \le \omega \le \pi$ $\frac{\text{rad.}}{\text{sample}}$

Note $\frac{1}{T} = 5000$





Suppose we band limit our signal to $\frac{1}{2}$ sampling frequency

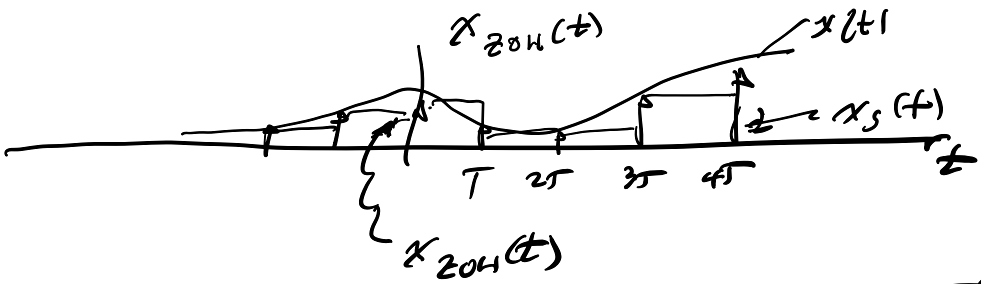


What happened?

1. We undersampled!
2. Recovery with ILP filter rejected original signal and passed the alias at 2 kHz

$\therefore$ should have sampled at $f_s > 6$ kHz

A real D/A convertor: Zero order hold (ZOH)



$$x_{ZOH}(t) = \underbrace{\text{comb}_T[x(t)]}_{x_s(t)} * \text{rect}\left(\frac{t - T/2}{T}\right)$$

$$\therefore \quad X_{ZOH}(f) = \underbrace{\frac{1}{T}\,\text{rep}_{\frac{1}{T}}[X(f)]}_{X_s(f)}\ T\,\text{sinc}(Tf)\,\underbrace{e^{-j2\pi \frac{T}{2} f}}_{\text{ignore}}$$

What does this look like?