ECE 438 Lecture 1/23/2023

LTI DT Systems

$$e^{j\omega n} \longrightarrow \boxed{\text{LTI DT System}} \longrightarrow H(\omega)\, e^{j\omega n}$$

This is part of frequency domain characterization – rest depends on DTFT

Time domain characterization for DT LTI systems

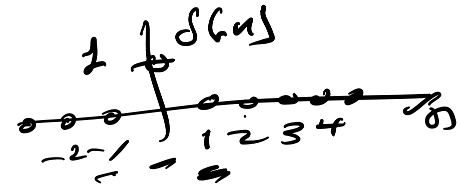
see Module 1.2.2 for background

$$\delta[n] \longrightarrow \boxed{\text{DT LTI System}} \longrightarrow h[n]$$

Example 1 2 pt. MA filter

$$y[n] = \frac{1}{2}\{x[n] + x[n-1]\}$$

let $x[n] = \delta[n]$



$n \neq 0$, $y[n] \equiv 0$

$$y[0] = \frac{1}{2}\{x[0] + x[-1]\}$$

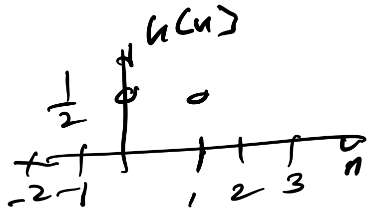
$$= \frac{1}{2}\{\delta[0] + \delta[-1]\}$$

$$= \frac{1}{2}$$

$$y[1] = \frac{1}{2}\{\delta[1] + \delta[0]\} = \frac{1}{2}$$

$n > 0 \quad y[n] = 0$

$$h[n] = \begin{cases} 0, & n < 0 \\ \frac{1}{2} & n = 0, 1 \\ 0, & n > 1 \end{cases}$$



Example 2

$$y[n] = x[n] - x[n-1] + \frac{1}{2} y[n-1] \quad \text{recursive}$$

Assume system is initially at rest, i.e.

$$y[n] = 0, \quad n \leq -1$$

$$n = -1 \quad h[n] = \delta[n] - \delta[n-1] + \frac{1}{2} h[n-1]$$

$$h[-1] = \quad -1] - \delta[-2] + \frac{1}{2} h[-2]$$

$$\Rightarrow h[-1] = 0$$

also $h[n] = 0, \quad n < 0$

$$n = 0 \quad h[0] = \overset{1}{\delta[0]} - \overset{0}{\cancel{\delta[-1]}} + \frac{1}{2} \overset{0}{\cancel{h[-1]}}$$

$$h[0] = 1$$

$$n = 1 \quad h[1] = \delta[1] - \delta[0] + \frac{1}{2} h[0]$$

$$= 0 - (+1) + \frac{1}{2} = -\frac{1}{2}$$

$$y[n] = x[n] - x[n-1] + \frac{1}{2} y[n-1]$$

$$n = 2 \quad h[2] = \delta[2] - \delta[1] + \frac{1}{2} h[1]$$

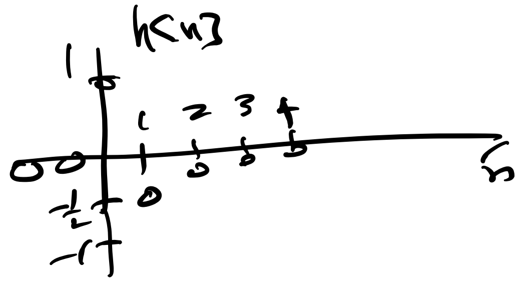
$$= 0 - 0 + \frac{1}{2}\left(-\frac{1}{2}\right) = -\frac{1}{4}$$

$$n = 3 \quad h[3] = -\frac{1}{8}$$

. . .

recognize the trend

$$h[n] = \begin{cases} 0, & n<0 \\ 1, & n=0 \\ -\left(\frac{1}{2}\right)^n, & n>0 \end{cases}$$



Example 3

$$y[n] = (x[n])^2$$

if $x[n] = \delta[n]$, $y[n] = h[n] = \delta[n]$

$\therefore$ can find unit sample response for any system.

Suppose we have a DT LTI system

by def. $x[n] = \delta[n]$

$\Rightarrow y[n] = h[n]$

By homogeneity,

$x[n] = a\,\delta[n] \Rightarrow y[n] = a\,h[n]$

a constant

By TI,

$x[n] = a\,\delta[n-k] \Rightarrow y[n] = a\,h[n-k]$

For any $x[n]$, have

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

sifting property of unit sample (see formulas)

Or

$$x[n] = \dots + \underbrace{x[-1]\delta[n-(-1)]}_{k=-1} + \underbrace{x[0]\delta[n]}_{k=0} + \overset{k=1}{x[1]\delta[n-1]} + \dots$$

Consider how each term goes through system

$x[-1]\delta[n+1] \rightarrow h[n+1]\,x[-1]$

$x[0]\delta[n] \rightarrow x[0]\,h[n]$

$x[1]\delta[n-1] \rightarrow x[1]\,h[n-1]$

Now apply superposition $\Rightarrow$ sum the above

$$y[n] = \dots + \overset{k=-1}{x[-1]\,h[n+1]} + \overset{k=0}{x[0]\,h[n]} + \underset{k=1}{x[1]\,h[n-1]} + \dots$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$ true for any DT LTI system

convolution sum

Step-by-step process for convolution of $x_1[n]$ & $x_2[n]$

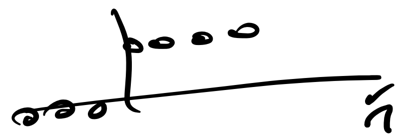
Step 1 Write formula for convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x_1[k]\,x_2[n-k]$$

$$= \sum_{k=-\infty}^{\infty} x_1[n-k]\,x_2[k]$$

Step 2 Write down $x_1[n]$ & $x_2[n]$

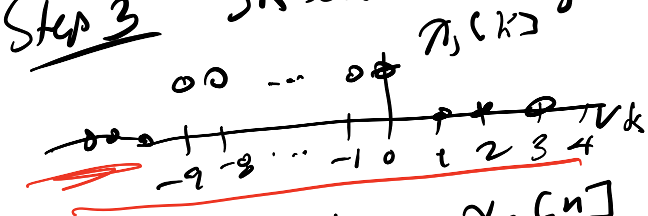
$x_1[n] = u[n+9] - u[n-1]$

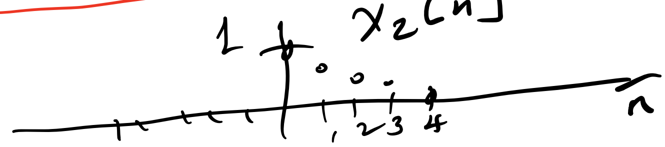


where $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{else} \end{cases}$

$x_2[n] = 2^{-n} u[n]$

Step 3 Sketch the signals

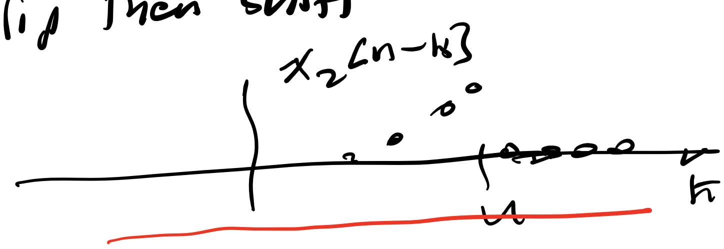




recall $y[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k]$

note that $x_2[n-k] = x_2[\underbrace{-}_{\text{flip}}(k \overbrace{-n}^{\text{shift}})]$

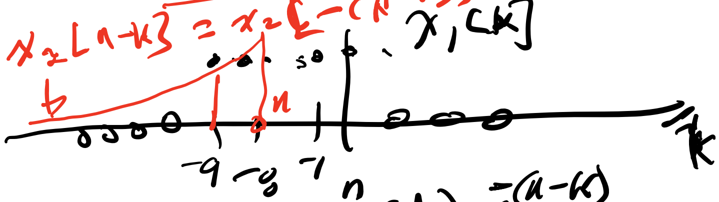
Flip then shift



Step 4 Determine cases & evaluate them separately

Case I $n < -9$ $y[n] = 0$

Case II $-9 \le n \le 0$



$$y[n] = \sum_{k=-9}^{n} (1)\, 2^{-(n-k)}$$

$$= 2^{-n} \underline{\sum_{k=-9}^{n} 2^{+k}}$$

recall

$$\sum_{k=0}^{N-1} z^k = \frac{1-z^N}{1-z}$$

let $l = k+9 \Rightarrow$ when $k=-9, l=0$

$\Leftarrow l = n+9$ when $k = n$

$k = l-9$

$$y[n] = 2^{-n} \sum_{l=0}^{n+9} 2^{(l-9)}$$

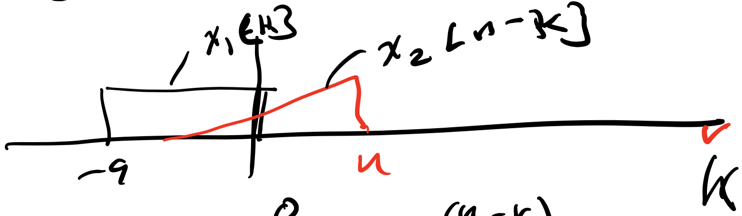
$$= 2^{-(n+9)} \sum_{l=0}^{n+9} 2^{l}$$

$$= 2^{-(n+9)} \frac{1-2^{n+10}}{1-2}$$

$$= 2^{-(n+9)} [2^{n+10} - 1]$$

$$= 2[1 - 2^{-n}/512]$$

Case III $\qquad n \geq 0$



$$y[n] = \sum_{k=-9}^{0} 2^{-(n-k)}$$

let $l = k+9 \Rightarrow k=-9 \Rightarrow l=0$

$k = l-9 \qquad k=0 \Rightarrow l=9$

$$y[n] = \sum_{k=0}^{9} 2^{-(n-(l-9))}$$