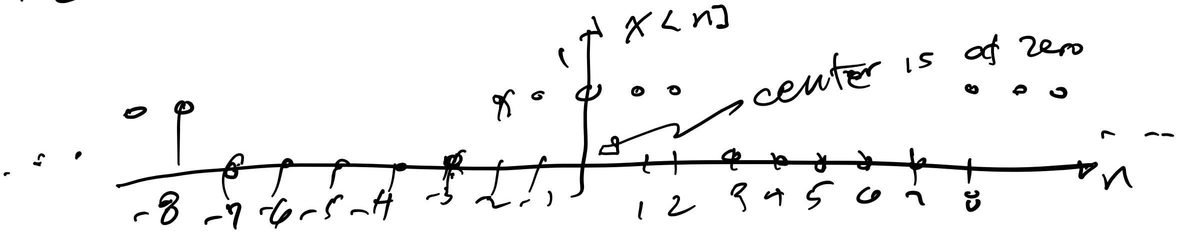
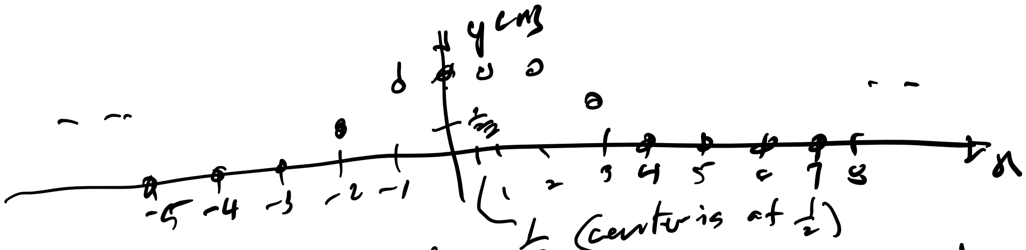
ECE 438 Lecture Friday 1/20/2023



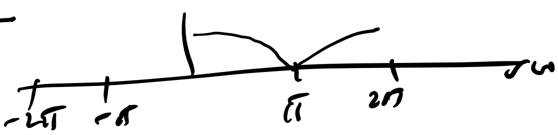
Considering response to $y[n] = \frac{1}{2}\{x[n] + x[n-1]\}$



$\underline{H(\omega) = \cos(\omega/2)\, e^{-j\omega/2}}$

$|H(\omega)| = \cos(\omega/2)$

LP filter!

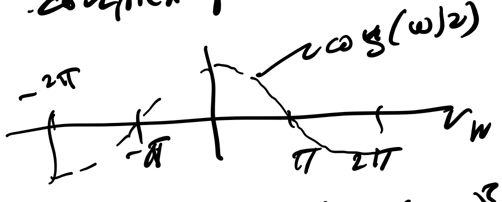


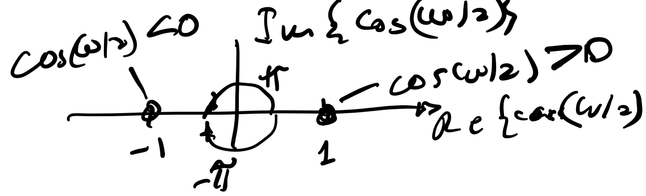
Delay is from the phase of frequency response

$\angle H(\omega) = \angle \cos(\omega/2) + \angle \underline{\underline{e^{-j\omega/2}}}$

$\angle \cos(\omega/2)$ is $\underline{\underline{\text{not}}}$ $\omega/2$

have to examine $\cos(\omega/2)$ in the complex plane

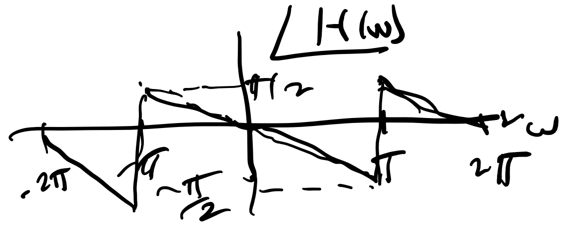




$$\angle \cos(\omega/2) = \begin{cases} 0, & \cos(\omega/2) > 0 \\ \pm\pi, & \cos(\omega/2) < 0 \end{cases}$$

So combining everything, we have

$$\angle H(\omega) = \begin{cases} -\omega/2, & \cos(\omega/2) > 0 \\ -\omega/2 \pm \pi, & \cos(\omega/2) < 0 \end{cases}$$



What does this tell us about response of the 2pt. MA to the period 10 square wave?

Consider $x[n] = \cos(\omega_0 n)$

$$= \frac{1}{2}\{e^{j\omega_0 n} + e^{-j\omega_0 n}\}$$

$$y[n] = \frac{1}{2}\{H(\omega_0)e^{j\omega_0 n} + H(-\omega_0)e^{-j\omega_0 n}\}$$

$$= \frac{1}{2}\{|H(\omega_0)|e^{j\angle H(\omega_0)}e^{j\omega_0 n} +$$

$$|H(-\omega_0)|e^{j\angle H(-\omega_0)}e^{-j\omega_0 n}\}$$

Since $|H(\omega_0)| = |H(-\omega_0)|$

Since $\angle H(\omega_0) = -\angle H(-\omega_0)$

$$y[n] = |H(\omega_0)| \frac{1}{2}\{e^{j(\omega_0 n + \angle H(\omega_0))} + e^{-j(\omega_0 n + \angle H(\omega_0))}\}$$

$$y[n] = |H(\omega_0)| \cos(\omega_0 n + \angle H(\omega_0))$$

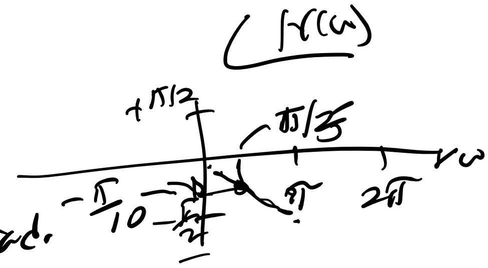
Consider again the special case where $x[n]$ is a square wave with period 10.

The fundamental frequency component

is $\cos\left(\frac{2\pi}{10} n\right) = \cos\left(\frac{\pi}{5} n\right)$

$$\Rightarrow \boxed{\omega_0 = \pi/5}$$

$$\left|H\left(\tfrac{\pi}{5}\right)\right| = 0.81$$



$$\angle H\left(\tfrac{\pi}{5}\right) = -\frac{\pi}{10} \text{ rad.}$$

$$y[n] = 0.81 \cos(\omega_0 n - \pi/10)$$

$$= 0.81 \cos\left(\tfrac{\pi}{5} n - \pi/10\right)$$

$$= 0.81 \cos\left(\tfrac{\pi}{5}\left(n - \tfrac{1}{2}\right)\right)$$

$\frac{1}{2}$ sample delay

Example 2

$$y[n] = \tfrac{1}{2}\{x[n] - x[n-1]\}$$

this is a primative differentiator!

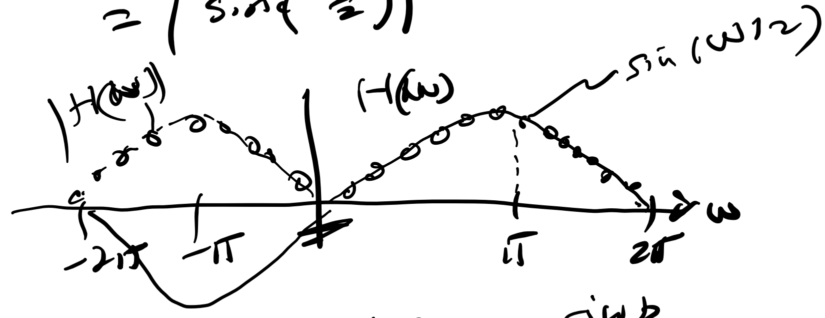
$$H(\omega) = \tfrac{1}{2}\{1 - e^{-j\omega}\}$$

$$= j \sin\left(\tfrac{\omega}{2}\right) e^{-j\omega/2}$$

$$|H(\omega)| = |j| \cdot |\sin(\tfrac{\omega}{2})| \cdot |e^{-j\omega/2}|$$

$$= |\sin(\tfrac{\omega}{2})|$$

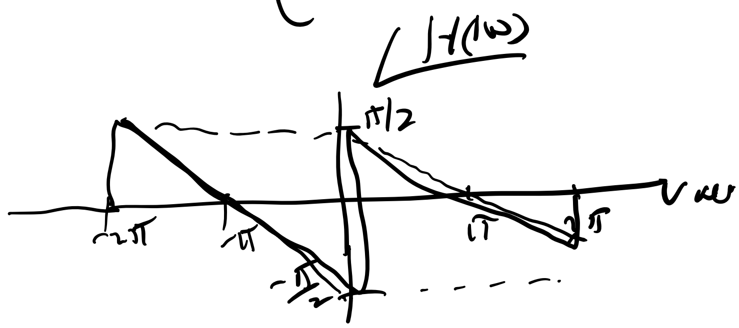
High pass filter



$$\angle H(\omega) = \angle j + \angle \sin(\tfrac{\omega}{2}) + \angle e^{j\omega/2}$$

$$= \frac{\pi}{2} - \omega/2 + \angle \sin(\tfrac{\omega}{2})$$

$$= \begin{cases} \frac{\pi}{2} - \omega/2, & 0 \le \omega \le 2\pi \\ \end{cases}$$



Example 3

$y[n] = x[n] \; x[n-1] - \underline{y[n-1]}$

recursive filter — also LTI

Find response to $e^{j\omega n}$

Assume $y[n] = H(\omega)e^{j\omega n}$

$e^{j\omega n}$ $\quad e^{j\omega(n-1)}$ $\quad e^{j\omega(n-1)}$

$$H(\omega)e^{j\omega n} = e^{u} - e \qquad = H(\omega) -$$

$$H(\omega)\{e^{j\omega n} + e^{j\omega(n-1)}\} = e^{j\omega n} - e^{j\omega(n-1)}$$

$$H(\omega) = \frac{e^{j\omega n} - e^{j\omega(n-1)}}{e^{j\omega n} + e^{j\omega(n-1)}}$$

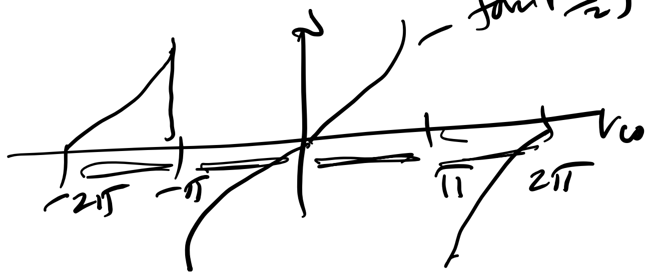
$$H(\omega) = \frac{\cancel{e^{j\omega n}}(1 - e^{-j\omega})}{\cancel{e^{j\omega n}}(1 + e^{-j\omega})}$$

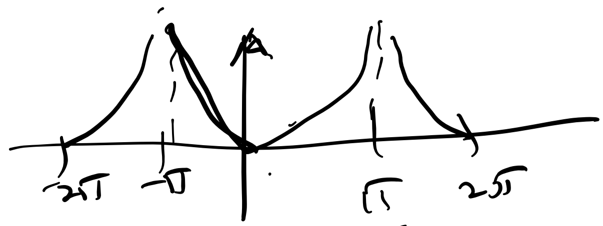
$$= \frac{(e^{+j\omega/2} - e^{-j\omega/2})\cancel{e^{-j\omega/2}}}{(e^{j\omega/2} + e^{-j\omega/2})\cancel{e^{-j\omega/2}}}$$

$$= \frac{j\cancel{2}\sin(\omega/2)}{\cancel{2}\cos(\omega/2)}$$

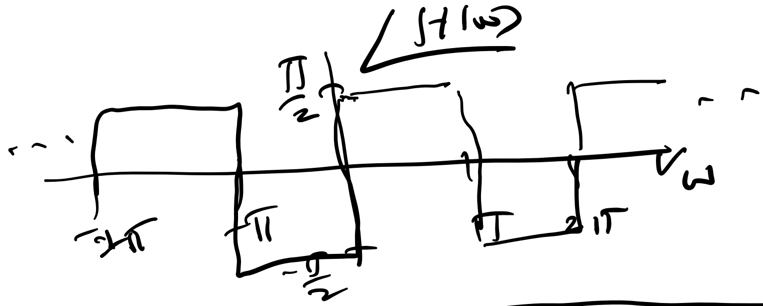
$$\boxed{H(\omega) = j\tan(\omega/2)}$$

$$|H(\omega)| = |\tan(\omega/2)|$$





$$\angle H(\omega) = \angle j + \angle \tan\left(\frac{\omega}{2}\right)$$



Time domain characterization of DT LTI system

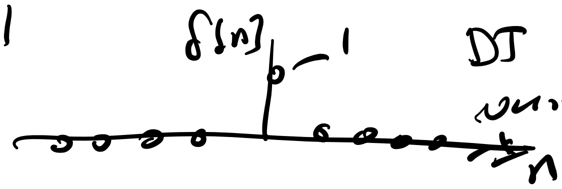
so far we have

$$x[n] = e^{j\omega_0 n} \longrightarrow \boxed{\text{DT LTI System}} \longrightarrow y[n] = H(\omega_0)e^{j\omega_0 n}$$

which is a frequency domain characterization

Consider response to $x[n] = \delta[n]$

recall



DT impulse or unit sample

define the response of <u>any</u> system to $x[n] = \delta[n]$ as $h[n]$ – unit sample response

$$\delta[n] \longrightarrow \boxed{\text{DT System}} \longrightarrow h[n]$$