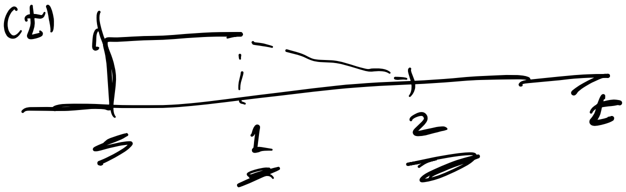
ECE 438 Lecture — Friday 13 January 2023

Signal transformations

$x_4(t)$

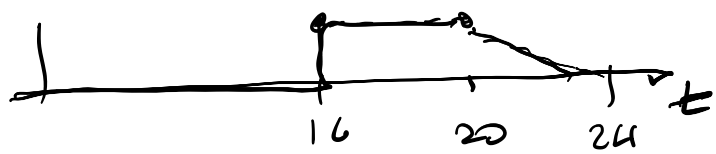


$x_5(t) = x_4(-t)$ time-reversal

$$x_6(t) = x_4\left(\frac{t}{4} - 4\right)$$

to see what this looks like, pick selected values for the argument.

(1) $\frac{t}{4} - 4 = 0 \Rightarrow t = 16$



(2) $\frac{t}{4} - 4 = 1 \Rightarrow t - 16 = 4 \Rightarrow t = 20$

(3) $\frac{t}{4} - 4 = 2 \Rightarrow t - 16 = 8 \Rightarrow t = 24$

standard form

$$x_4\left(\frac{t}{4} - 4\right) = x_4\left(\frac{t - 16}{4}\right)$$

shifted amount

scaling amount

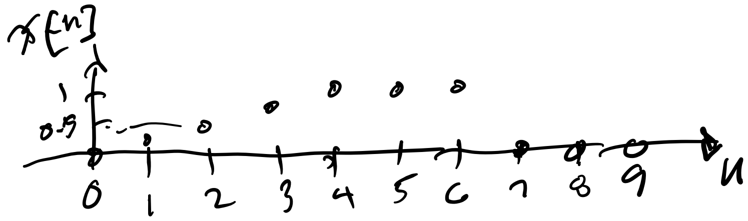
DT signal transformations

2 special cases

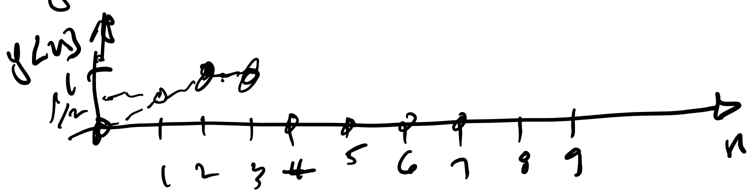
① Downsampling

$$x[n] \rightarrow \boxed{D\downarrow} \rightarrow y[n] \qquad y[n] = x[Dn]$$

D - integer



$y[n] = x[2n] \quad D=2$

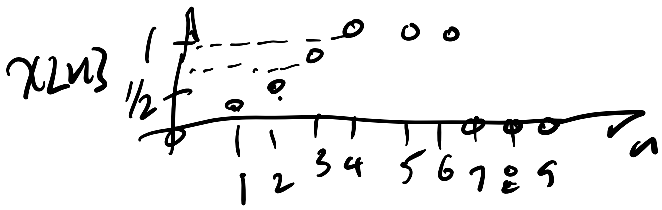


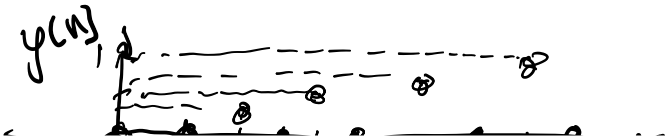
took every other sample & dropped the samples for odd argument in $x[n]$

$\Rightarrow$ signal is compressed

② Upsampler

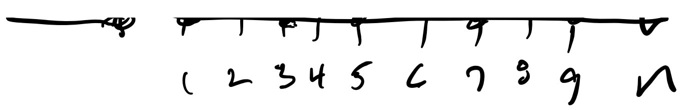
$$x[n] \rightarrow \boxed{D\uparrow} \rightarrow y[n] \qquad y[n] = \begin{cases} x[n/D], & n/D \text{ is an integer} \\ 0, & \text{else} \end{cases}$$





stretching signal

D = 2



Inserted 1 zero between every sample in $x[n]$

Why would we do this?

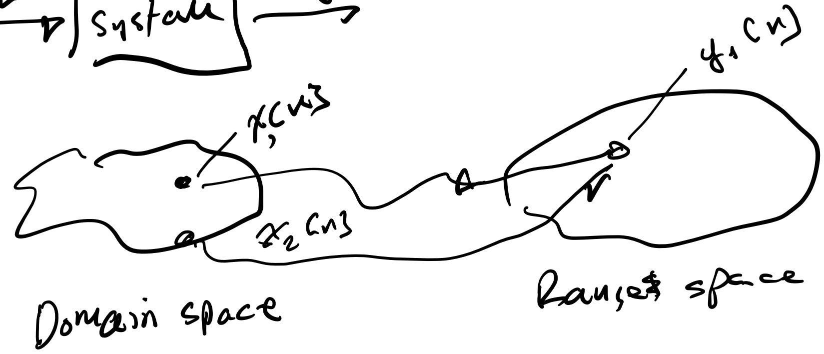
Upsampling is an analytical tool

$$x[n] \rightarrow \boxed{DA} \xrightarrow{y[n]} \boxed{\text{Digital filter}} \rightarrow z[n]$$

interpolator

Module 1.2

$$x[n] \rightarrow \boxed{\text{System}} \rightarrow y[n]$$



Can have $x_1[n] \neq x_2[n]$, but $y_1[n] = y_2[n]$

Two important system properties

① linearity : homogeneity
superposition

def A system is linear $\iff$

— we have

$$\begin{cases} x_1[n] \xrightarrow{\text{Sys}} y_1[n] \\ x_2[n] \xrightarrow{\text{Sys}} y_2[n] \end{cases}$$

then constant

$$x_3[n] = a_1 x_1[n] + a_2 x_2[n]$$

produces

$$y_3[n] = a_1 y_1[n] + a_2 y_2[n]$$

i.e. $x_3[n] \xrightarrow{\text{Sys}} y_3[n]$

Ex. 1 $\quad y[n] = \frac{1}{2}\{x[n] + x[n-1]\}$ 2-pt. moving average (MA)

Is this linear?

Hypothesize that system is linear
⇒ have to prove it for all possible inputs

let $x_3[n] = a_1 x_1[n] + a_2 x_2[n]$

$$y_3[n] = \frac{1}{2}\{x_3[n] + x_3[n-1]\}$$

$$= \frac{1}{2}\{(a_1 x_1[n] + a_2 x_2[n]) + (a_1 x_1[n-1] + a_2 x_2[n-1])\}$$

$$= a_1 \frac{1}{2}\{x_1[n] + x_1[n-1]\} + a_2 \frac{1}{2}\{x_2[n] + x_2[n-1]\}$$

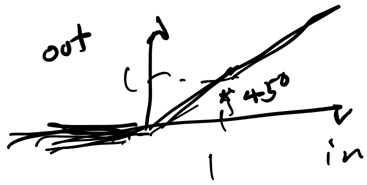
$= \underline{a_1 y_1[n] + a_2 y_2[n]}$

$\therefore$ system $\underline{\text{is}}$ linear

Example 2

$$y[n] = \begin{cases} x[n], & \text{if } x[n] > 0 \\ \underline{0}, & \text{else} \end{cases}$$

zero memory nonlinearity (ZMNL)



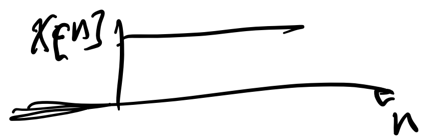
half-wave rectifier

hypothesize that system is nonlinear

$\Rightarrow$ need to find only $\underline{\underline{\text{one}}}$ counterexample

pick something as simple as possible!

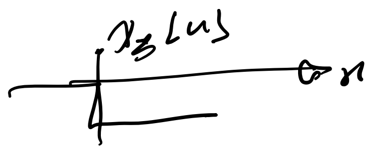
$x_1[n] = u[n]$



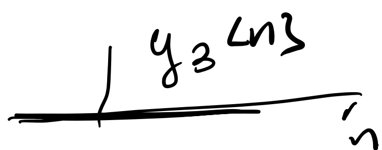
$a_1 = -1, \; a_2 = 0$

$y_1[n] = u[n]$

$x_3[n] = -1 \; u[n]$



$$y_3[n] \equiv 0 \neq a_1 y_1[n]$$



$\therefore$ system is $\underline{\text{not}}$ linear

System Property No. 2 $\Rightarrow$ time-invariance (TI)

def. If $x_1[n] \xrightarrow{sys} y_1[n]$

then system is TI if $x_2[n] = x_1[n-n_0]$

and $y_2[n] = y_1[n-n_0]$ n_0 - integer

Examples

① 2 pt. MA   let $x_1[n] \longrightarrow y_1[n]$

let $x_2[n] = x_1[n-n_0]$

then $y_2[n] = \frac{1}{2}\{x_2[n] + x_2[n-1]\}$

$= \frac{1}{2}\{x_1[n-n_0] + x_1[n-1-n_0]\}$

$= \frac{1}{2}\{x_1[n-n_0] + x_1[n-n_0-1]\}$

$= y_1[n-n_0]$

$\therefore$ 2 pt. MA is time-invariant

② $y[n] = \begin{cases} x[n], & x[n] > 0 \\ 0, & \text{else} \end{cases}$ Note that sys definition does not depend on time? $\Rightarrow$ probably TI

let $x_1[n] \longrightarrow y_1[n]$

then let $x_2[n] = x_1[n-n_0]$

$y_2[n] = \begin{cases} x_1[n-n_0], & x_1[n-n_0] > 0 \\ 0, & \text{else} \end{cases}$

$= y_1[n-n_0]$ $\therefore$ sys is TI

③ $y[n] = \begin{cases} x[n], & n \geq 0 \\ 0, & \text{else} \end{cases}$