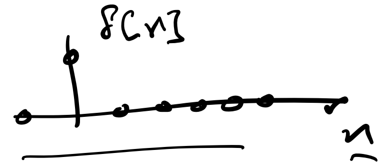
ECE 438 Lecture 11 Jan. 2023

Modules 1.1.2 & 1.1.3

Special signals

1. DT impulse $\delta[n] = \begin{cases} 1, & n=0 \\ 0, & \text{else} \end{cases}$



2. DT unit step $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{else} \end{cases}$

Signal metrics for $x[n]$

1. magnitude $M_x = \max_n |x[n]|$
2. Area $A_x = \sum_n x[n]$

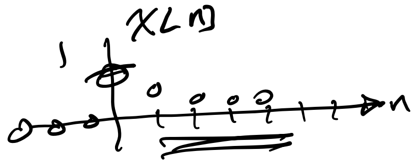
become familiar with summation notation

$\sum_n$ n - index over which summation is performed

if limits are not shown, assume that summation goes from $-\infty$ to ∞

i.e. $\sum_n \equiv \sum_{n=-\infty}^{\infty}$

Ex 1 $x[n] = e^{-n/4} u[n]$



$$M_x = \max_n |x[n]| = 1$$

$$A_x = \sum_{n=-\infty}^{\infty} e^{-n/4} u[n] = \boxed{\sum_{n=0}^{\infty} e^{-n/4}}$$

Formula: geometric series

$$\sum_{n=0}^{N-1} z^n = \frac{1-z^N}{1-z}$$ z - possibly complex number

$$A_x = \sum_{n=0}^{\infty} = \frac{1-(e^{-1/4})^N}{1-e^{-1/4}} \quad N \to \infty$$

$z = e^{-1/4}$

$$\lim_{n\to\infty} e^{-n/4} ? = 0$$

$$A_x = \frac{1}{1-e^{-1/4}} = 4.522$$

③ energy $E_x = \sum_n |x[n]|^2$

ex. 1.:

$$= \sum_{n=0}^{\infty} e^{-n/2} = \frac{1}{1-e^{-1/2}} = 2.542$$

by geometric series

$-1/2$

Example 2

$$x_2[n] = \cos(\underline{0.25\pi n})$$

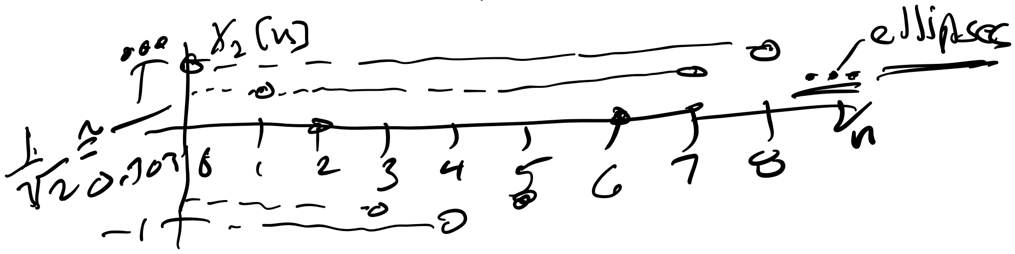
What does it look like?

standard form for sinusoids $x_2[n] = \cos(2\pi n/P)$

P - period

Note: P need not be an integer

here: $x_2[n] = \cos(2\pi n/8)$



$$M_{x_2} = \max_n |x[n]| = 1$$

$$A_{x_2} = \lim_{M \to \infty} \sum_{n=-M}^{M} x_2[n] \quad \text{undefined}$$

sum does not converge

3. average value

$$x_{avg} = \lim_{M \to \infty} \frac{1}{2M+1} \sum_{n=-M}^{M} x[n]$$

$$= \frac{\left[\lim_{M \to \infty} \sum_{n=-M}^{M} x[n]\right]}{\left[\lim_{M \to \infty} 2M+1\right]}$$

For E_x:

- numerator does not converge, but it's bounded between -1 & 1
- denominator goes to ∞

$\therefore \; {}_2x_{avg} = \underline{\underline{0}}$

Considering example 2, $x_2[n] = \cos((2\pi/8) n)$

$$E_x = \lim_{M \to \infty} \sum_{n=-M}^{M} |x[n]|^2 \text{ doesn't converge}$$

④ $$P_x = \lim_{M \to \infty} \frac{1}{2M+1} \sum_{n=-M}^{M} |x[n]|^2$$

Fact 1: Recall that $x_2[n]$ is periodic with period $P = 8$

For any signal $x[n]$ periodic with period P, have that

$$\sum_{n=n_0}^{n_0+P-1} x[n] = \sum_{n=0}^{P-1} x[n] \text{ for any integer } n_0$$

Fact 2: P_x can be calculated by summing over any period P ~~for~~ a signal with period P

$$P_x = \frac{1}{P} \sum_{n=0}^{P-1} \left[\cos(2\pi/8\, n)\right]^2$$

How do we evalaate this sum?

Trick No. 1 $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$

$$P_x = \frac{1}{8}\sum_{n=0}^{7}\left(\frac{1}{2}\right) + \frac{1}{8}\sum_{n=0}^{7}\cos\left(\frac{2\pi}{4}n\right)$$

$$= \frac{1}{2} + 0$$

summed over 2 periods $\Rightarrow 0$

Trick No. 2

(5) $x_{rms} = \sqrt{P_x}$

$x_{rms} = \sqrt{\frac{1}{2}}$

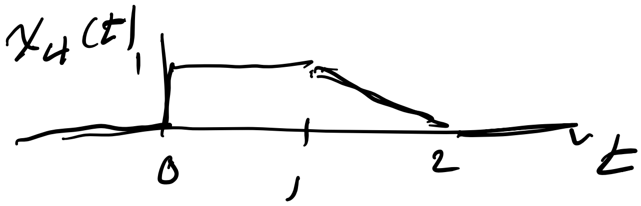
example cosine that is not periodic

$x_3[n] = \cos(n) = \cos(2\pi n / 2\pi)$

period of = 6.28

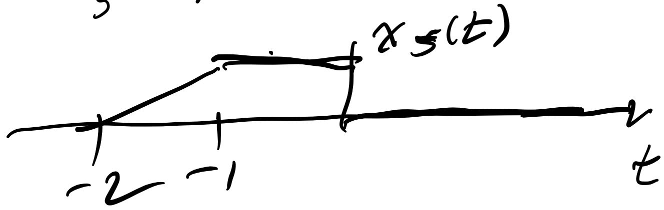
Module 1.1.3 Signal transformations

use CT here to make visualization easier

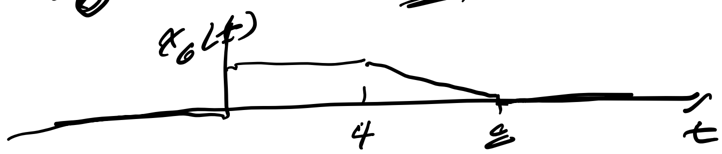


time-reversal

① $x_5(t) = x_4(-t)$



② $x_6(t) = x_4(t/4)$ time-scaling



③ $x_7(t) = x_4(\frac{t}{4} - 4)$ what ~~dos~~ does this look like?