ECE 438 Lecture   Feb. 3, 2023

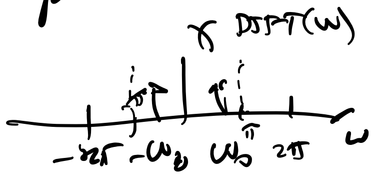
Relation between DTFT & CTFT

$$X^{DTFT}(\omega) = X_s^{CTFT}(f)\Big|_{f = \frac{\omega}{2\pi} f_s}$$

or

$$X_s^{CTFT}(f) = X^{DTFT}(\omega)\Big|_{\omega = \frac{f}{f_s} 2\pi}$$

previous example: $x^{CT}(t) = \cos(2\pi f_0 t)$



$$X^{DTFT}(\omega) = \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

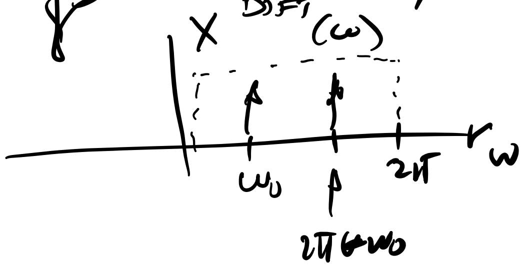
$$-\pi \le \omega \le \pi$$

or

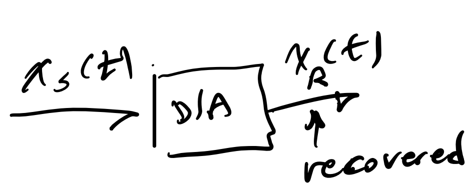
equivalent

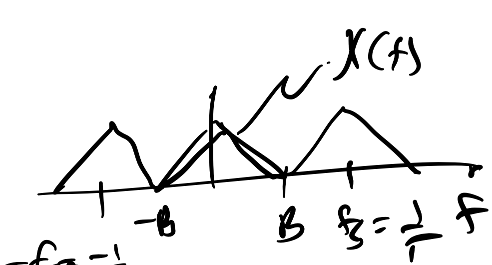
$$X^{DTFT}(\omega) = \pi\,[\delta(\omega - \omega_0) + \delta(\omega - 2\pi + \omega_0)]$$

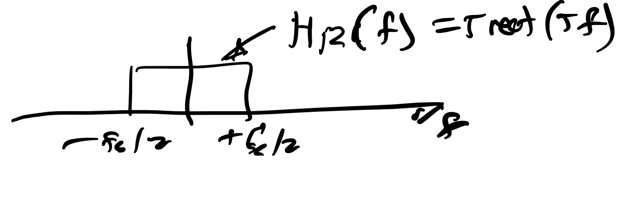
$$0 \le \omega \le 2\pi$$



D/A converter





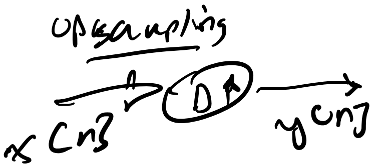


Module 1.4.2.1 – Relation between CTFT & DTFT

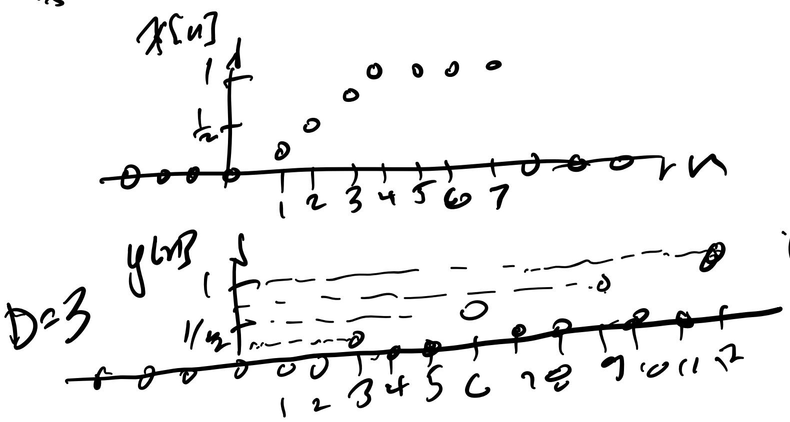
would like to avoid having to use an ideal lowpass analog filter with a sharp cutoff at $f = f_s/2$

Upsampling & Downsampling in DTFT domain

Module 1.4.3 – not covered on Exam No. 1.



$$y[n] = \begin{cases} x[n/D], & \text{when } n/D \text{ is an integer} \\ 0, & \text{else} \end{cases}$$



Insert two zeros between every sample in x[n]

Note: ① we have not discarded any information
⇒ no aliasing

② we have stretched the signal in the time domain ⇒ will compress the spectrum in the frequency domain

2. Relatio $-j\omega n$

$$Y(\omega) = \sum_{n}' x[n/D] e^{\ldots} =$$

(n/D - is an integer)

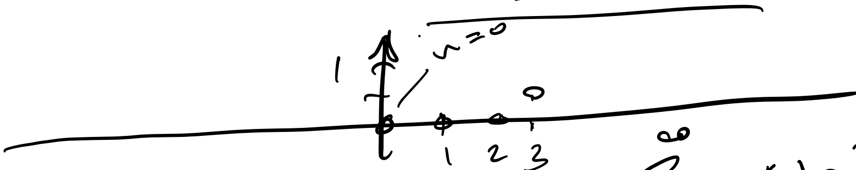
what are we looking for?

want to find something in terms of $X(\omega)$

so $X(\omega) = \sum_{n} x[n] e^{-j\omega n}$

let $m = n/D \Rightarrow n = mD$

have $Y(\omega) = \sum_{m=-\infty}^{\infty} x[m] e^{-j\omega m D} = X(\omega D)$



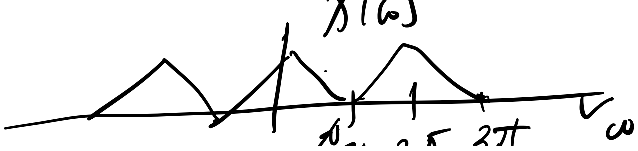
original definition of $Y(\omega) = \sum_{n=-\infty}^{\infty} x(n/D) e^{-j\omega n}$
when n/D is an integer

$$Y(\omega) = \ldots + \underbrace{x(-1)}_{n=-3} e^{+j\omega 3} + \underbrace{x(0)}_{n=0} e^{-j\omega 0} + \underbrace{x(1)}_{n=3} e^{-j\omega 3} + \ldots$$

$$Y(\omega) = \ldots + \underbrace{x[-1]}_{m=-1} e^{+j\omega 3} + \underbrace{x[0]}_{m=0} e^{-j\omega 0} + \underbrace{x[1]}_{m=1} e^{-j\omega 3}$$

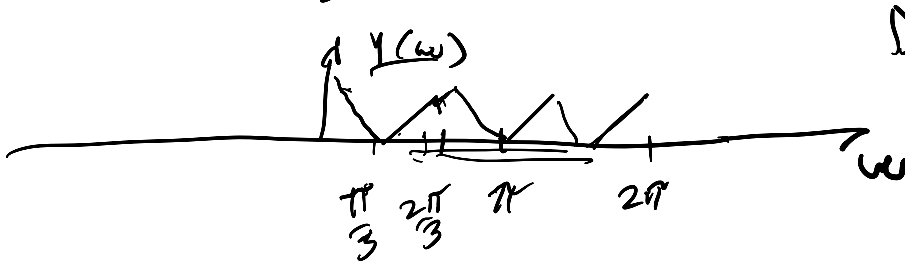
final result is $Y(\omega) = X(\omega D)$

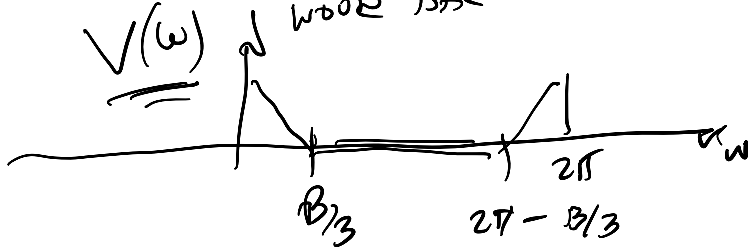
$X(\omega)$

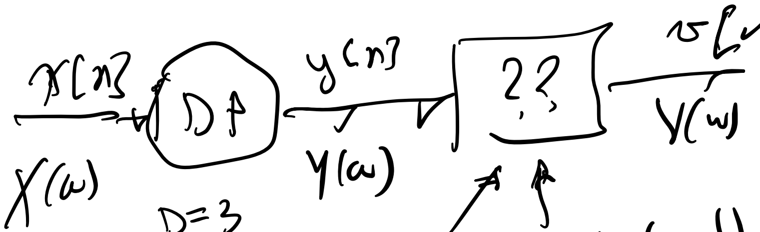


$B = \pi$

$D = 3$

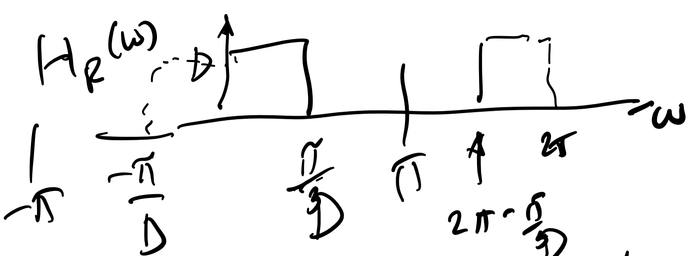






$H_R(\omega)$ what should this be? (A ILP digital filter

This will be an interpolator.



$D = 3$

What is happening in time domain?

$$v[n] = \underline{h_R[n]} * y[n]$$

$$h_R[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_R(\omega) e^{j\omega n}\, d\omega$$

$$= \frac{D}{2\pi}\int_{-\pi/D}^{\pi/D} e^{j\omega n}\, d\omega$$

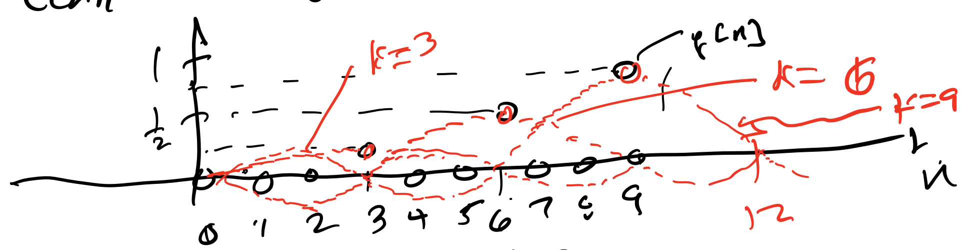
$$\int e^{j\omega n}\, d\omega = \frac{1}{jn} e^{j\omega n}$$

$$h_R[n] = \frac{D}{2\pi}\left\{\frac{e^{j\pi/D\, n}}{jn} - \frac{e^{-j\pi/D\, n}}{jn}\right\}$$

$$= \frac{\overbrace{\frac{1}{j2}\left\{e^{j\pi/D\, n} - e^{-j\frac{\pi}{D} n}\right\}}^{\sin\left(\frac{\pi}{D} n\right)}}{\pi n/D}$$

$$= \underline{\mathrm{sinc}\,(n/D)}$$

recall $\mathrm{sinc}(t) = \sin(\pi t)/(\pi t)$



$$v[n] = h_R[n] * y[n]$$

$$= \sum_{k=-\infty}^{\infty} y[k]\, \mathrm{sinc}\left(\frac{n-k}{D}\right)$$

note:

when $n = Dl$ — l integer, $v[n] = y[n/D]$

otherwise, we smoothly interpolate between

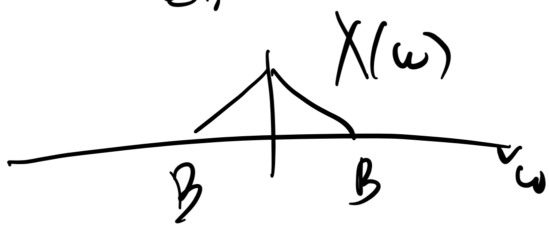
$y[n]$

nonzero samples y(n)

Shannon sampling expansion

earlier derived by Whittaker and Kotelnikov

so now it is known as the WKS (Whittaker-Shannon-Kotelnikov) sampling expansion



B - highest frequency in radians/sampling