ECE 438 Lecture — 1 Feb 2023

Announcements:

- Office Hour today at 3:30p EST
- HW #2 due at 11:59p EST via Gradescope

Review

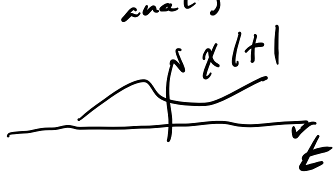
- $$\mathrm{rep}_T[x(t)] = \sum_{k=-\infty}^{\infty} x(t-kT)$$
- $$\mathrm{comb}_T[x(t)] = \sum_{k=-\infty}^{\infty} x(kT)\,\delta(t-kT)$$

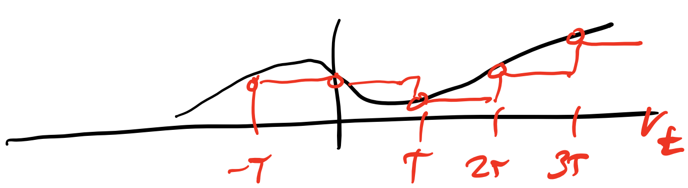
$$\mathrm{rep}_T[x(t)] \xrightarrow{CTFT} \frac{1}{T}\,\mathrm{comb}_{\frac{1}{T}}[X(f)] \Leftarrow \text{Fourier series}$$

$$\mathrm{comb}_T[x(t)] \xrightarrow{CTFT} \frac{1}{T}\,\mathrm{rep}_{\frac{1}{T}}[X(f)] \Leftarrow \text{sampling}$$

ZOH D/A convertor

$$\underset{\uparrow \text{ analog}}{x_{ZOH}(t)} = \sum_{k=-\infty}^{\infty} x[n]\,\mathrm{rect}\left(\frac{t-T/2}{T}\right)$$

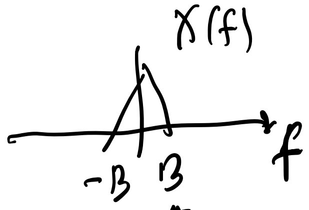




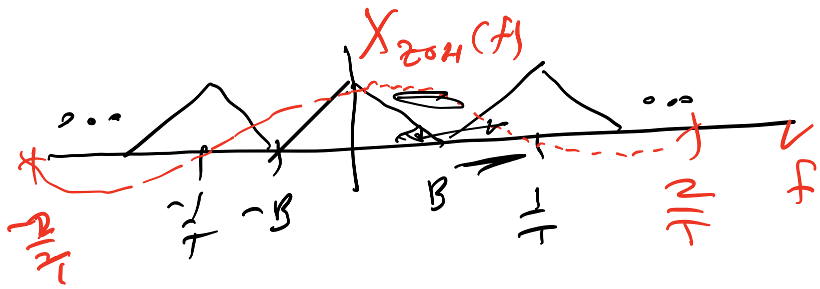
$$x_{ZOH}(t) = \mathrm{comb}_T[x(t)] * \mathrm{rect}\left(\frac{t-T/2}{T}\right)$$

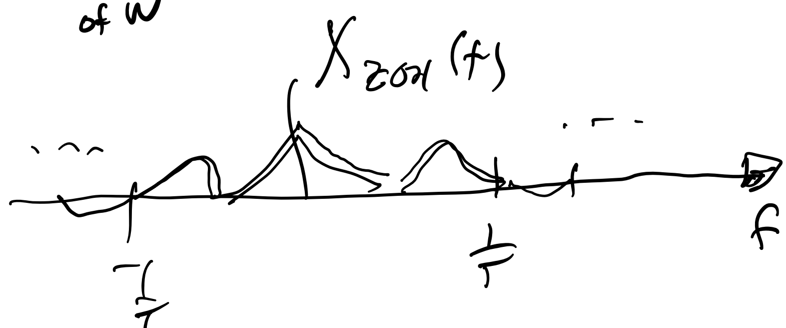
$$X_{ZOH}(f) = \frac{1}{T}\,\text{rep}_{\frac{1}{T}}[X(f)]\; T\; \text{sinc}(Tf)\; e^{-j2\pi f T/2}$$

$\uparrow$ ignore



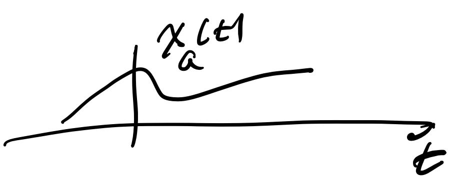
use this instead of W



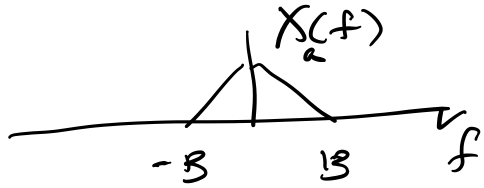


New topic: Relation between CTFT & DTFT

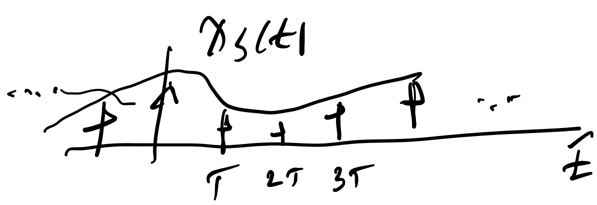
Time

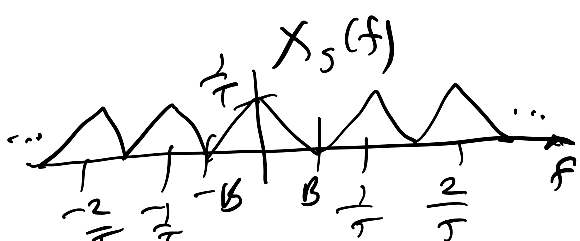


Frequency



$$x_s(t) = \text{comb}_T[x_a(t)]$$





let's naively take CTFT of $x_s(t)$:

$$x_s(t) = \sum_{k=-\infty}^{\infty} x_a(kT)\,\delta(t-kT)$$

$$X_s(f) = \int_{-\infty}^{\infty} x_s(t)\, e^{-j2\pi f t}\, dt$$

$$X_s(f) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_a(kT)\,\delta(t-kT)\, e^{-j2\pi f t}\, dt$$

interchange summation and integration

$$X_s(f) = \sum_{k=-\infty}^{\infty} x_a(kT) \underbrace{\int_{-\infty}^{\infty} \delta(t-kT)\, e^{-j2\pi f t}\, dt}$$

$$X_s(f) = \sum_{k=-\infty}^{\infty} x_c(kT)\, e^{-j2\pi f kT}$$

note that $x_c(kT) = x_d[k]$

$$X_s(f) = \sum_{k=-\infty}^{\infty} x_d[k]\, e^{j2\pi f kT}$$

recognize what this is !

What was I looking for?

want to relate CTFT to DTFT

would like to see $X_s^{DT}(\omega) = \varphi\{X_s^{CT}(f)\}$

What does DTFT look like?

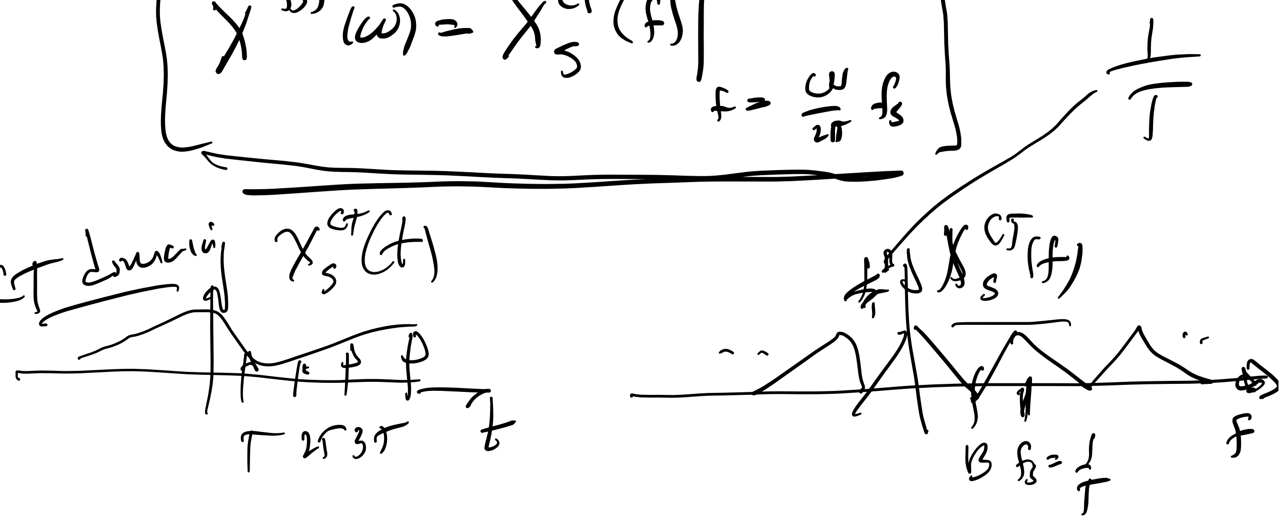
$$X^{DT}(\omega) = \sum_{k=-\infty}^{\infty} x_d[k]\, e^{j\omega k}$$

let $\omega = 2\pi f T$ note that $T = \frac{1}{f_s}$ — sampling frequency

$$\omega = 2\pi \frac{f}{f_s} \Rightarrow f = \frac{\omega}{2\pi} f_s$$

$$\boxed{X^{DT}(\omega) = X_s^{CT}(f)\Big|_{f = \frac{\omega}{2\pi} f_s}}$$

CT domain



DT domain

f_s

Example

$$x_a^{CT}(t) = \cos(2\pi f_0 t) \quad X_a^{CT}(f) = \frac{1}{2}\{\delta(f-f_0) + \delta(f+f_0)\}$$

$$x_d^{DT}[n] = x_a^{CT}(nT)$$

$$= \cos(2\pi f_0 nT)$$

$$X_s^{CT}(f) = \frac{1}{T}\,\text{rep}_{\frac{1}{T}}[X_a^{CT}(f)] \qquad f_s = \frac{1}{T}$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} \frac{1}{2}\{\delta(f - f_0 - \frac{k}{T}) + \delta(f + f_0 - \frac{k}{T})\}$$

Know that

$$\cos(\omega_0 n) \xrightarrow{DTFT} \pi\{\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\}$$

$$-\pi \le \omega \le \pi$$

Let $f = \frac{\omega}{2\pi} f_s$

$$X_d^{DT}(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \frac{1}{2}\left\{\delta\left(\frac{\omega}{2\pi f_s} - f_0 - \frac{k}{T}\right) + \delta\left(\frac{\omega}{2\pi f_s} + f_0 - \frac{k}{T}\right)\right\}$$

Use relationship

$$\delta(ax+b) = \frac{1}{|a|}\delta(x + b/a) \qquad a = \frac{f_s}{2\pi}$$

$$= \cancel{\frac{1}{T}} \sum_{k=-\infty}^{\infty} \frac{1}{\cancel{2}}\left\{\cancel{2\pi f_s}\,\pi\,\delta\left(\omega - \frac{f_0 2\pi}{f_s} - \frac{k 2\pi f_s}{f_s}\right) + \cancel{2\pi f_s}\,\pi\,\delta\left(\omega + \frac{f_0 2\pi}{f_s} - \frac{k 2\pi f_s}{f_s}\right)\right\}$$

$$= \sum_{k=-\infty}^{\infty} \pi\left\{\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)\right\}$$

This concludes coverage for Exam 1 and HW1, HW2, HW3