ECE 438 lecture 7 April 2023

(Friday)

Announcements:

HW #9 is posted. Due Wednesday 15 April on Gradescope 11:59p EDT

Linear Predictive Coding of Speech (LPC)

check wikipedia for history of LPC

Idea: Rather than transmit samples of the speech waveform, we transmit a set of parameters

Notes are from Module 4.3 – prepared by Prof. Michael Zoltowski.

Divide speech waveform into non-overlapping frames, each containing N symbols

Define a window $w[m] \neq 0$, only for $0 \leq m \leq N-1$

Let $s_n[m] = w[m]\underbrace{s[m+n]}_{\text{speech waveform}}$

m – time index

n – frame location

Define a predictor

$$\hat{s}_n[m] = \sum_{k=1}^{p} \alpha_k \overbrace{s_n[m-k]}^{\text{known}}$$

parameters of predictor are α_k, $k = 1, \dots, p$

$s_n[m] \neq 0$, for $0 \le m \le N-1$

but $\hat{s}_n[m] \neq 0$, for $0 \le m \le N+p-1$

Goal: choose coefficients α_k, to minimize

$$E_n = \sum_{m=0}^{N+p-1} f_n^2[m]$$

$$f_n[m] = s_n[m] - \hat{s}_n[m]$$

recall $e[m]$ is excitation

Showed last class that if α_k's (known) match

a_k's from speech model, then

$$f_n[m] = \underbrace{G}_{\text{gain}}\,\underbrace{e[m]}_{\text{excitation}}$$

This allows us to estimate pitch period

(not covered in ECE 438)

Also have to estimate the gain G, also

will not cover this in ECE 438

Three epochs for estimation error:

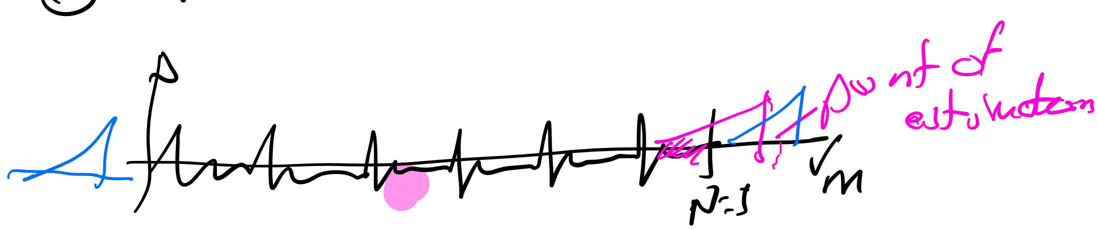
(a) $0 \le m \le p$

so we are missing some data ⇒ larger errors

(b) $p \le m \le N-1$

have a full set of data ⇒ best possible prediction

(c) $N \le m \le N+p-1$



trying to predict zero from non-zero data ⇒ error will be larger

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There are two methods to solve the α_k's !

(1) covariance method :

$$E = \sum_{m=p}^{N-1} f_n^2[m]$$

will not ... a more accurate

derivation's in class

See notes:

- more computation ($O(p^3)$)
- Can be unstable, i.e. poles of predictor can be outside unit circle in z-plane

(2) auto-correlation method:

$$E = \sum_{m=-\infty}^{\infty} f_n^2[m]$$

note that limits are actually $m=0,\dots,N+p-1$

- less accurate
- less computation ($O(p^2)$)

$$E = \sum_{m=-\infty}^{\infty} f_n^2[m]$$

$$= \sum_{m=-\infty}^{\infty} \left\{ \left(s_n[m] - \sum_{k=1}^{p} \alpha_k s_n[m-k] \right)^2 \right\}$$

Goal: find optimal α_k's $k=1,\dots,p$

Consider fixed l, & then compute

$$\frac{\partial E}{\partial \alpha_l} = \sum_{m=-\infty}^{\infty} \frac{\partial}{\partial \alpha_l} \{ \quad \}$$

chain rule for differentiation

$$= \sum_{m=-\infty}^{\infty} 2 \left(s_n[m] - \sum_{k=1}^{p} \alpha_k s_n[m-k] \right) \frac{\partial}{\partial \alpha_l} (\cdot)$$

$$= \sum_{m=-\infty}^{\infty} 2(\cdot)\, s_n[m-l]$$

let $\frac{\partial E}{\partial \alpha_l} = 0 \Rightarrow$

$$\underline{\sum_{m=-\infty}^{\infty} s_n[m] s_n[m-l]} = \sum_{\underline{k=1}}^{p} \alpha_k \underline{\sum_{m=-\infty}^{\infty} s_n[m-k] s_n[m-l]}$$

On l.h.s, let $m' = m - l \Rightarrow m = m' + l$

$$\sum_{m=-\infty}^{\infty} s_n[m] s_n[m-l] = \sum_{m'=-\infty}^{\infty} s_n[m'+l] s_n[m']$$

define $\underline{R_n[l]} = \sum_{m=-\infty}^{\infty} s_n[m] s_n[m+l]$

just showed that $\overline{R_n[l] = R_n[-l]}$

let $m' = m - l \Rightarrow m = \underline{m' + l}$

$$\sum_{m=-\infty}^{\infty} s_n[\underline{m-k}] s_n[m-l] = \sum_{m'=-\infty}^{\infty} s_n[m'+l-k] s_n[m']$$

$$= R_n[l-k]$$

have is:

$$R_n[l] = \sum_{k=1}^{p} \alpha_k \overbrace{R_n[l-k]}^{R_n[k-l]} \quad \text{where}$$

$l = 1, \ldots, p$

$0, \ldots, p-1$

have p linear equations in unknown α_l's

$$\begin{bmatrix} R_n[0] \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} R_n[0] & R_n[1] & R_n[2] & \cdots & R_n[p-1] \\ R_n[1] & R_n[0] & R_n[1] & \cdots & R_n[p-2] \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \vdots \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \vdots \\ R_n[p] \end{bmatrix}}_{\vec{R}} \quad \underbrace{\begin{bmatrix} & \ddots & & \\ R_n[p-1] & R_n[p-2] & \cdots & R_n[0] \end{bmatrix}}_{\underline{R}} \underbrace{\begin{bmatrix} \vdots \\ \alpha_p \end{bmatrix}}_{\vec{\alpha}}$$

so we have

$$\vec{R} = \underline{R}\,\vec{\alpha}$$

note the special structure of this matrix:

① $\underline{R} = \underline{R}^T$ – symmetric

② each row is a shift version of the row above it
always have $R_n[0]$ on the diagonal

① & ② ⇒ Matrix is Toeplitz

thus equations can be solved very efficiently
using a method called Levinson-Durbin
recursion

Final error can be expressed as

$$E_n = R_n[0] - \sum_{k=1}^{p} \alpha_k R_n[k] \quad \text{easy to show}$$

method is guaranteed to produce a stable
predictor, i.e. poles in z-plane are inside
the unit circle

How many poles do we need?