ECE 438 Lecture Monday 3 April 2023

Announcements:

* Office Hours to : 2:30p EDT
  4:00p EDT

* Quiz – covering material from HW No. 7
  - available at p EDT
  - have 30 minutes to work on it
  - must upload solution by 11:59p EDT

$$\tilde{S}(\omega_0, n) = \sum_m \underline{s[m]} w[n-m] e^{-j\omega_0 m}$$

$w[-(m-n)]$ ω_0

center of window

treat n as time variable, sample frequency at $\omega_r = \frac{2\pi r}{N}$, $r = 0, 1, \ldots, N-1$

note N – no. frequency samples not length of window $w[n]$

$$S_r[n] = \sum_k \underline{s[k]} \, \underline{h[n-k]} \, \underline{e^{-j\frac{2\pi k r}{N}}}$$

note: we are switching notation from $w[n] \to h[n]$ – think of it as a filter

use commutative property of convolution:

$$s_r[n] = \sum_k s[n-k]h[k]e^{-j2\pi\frac{(n-k)r}{N}}$$

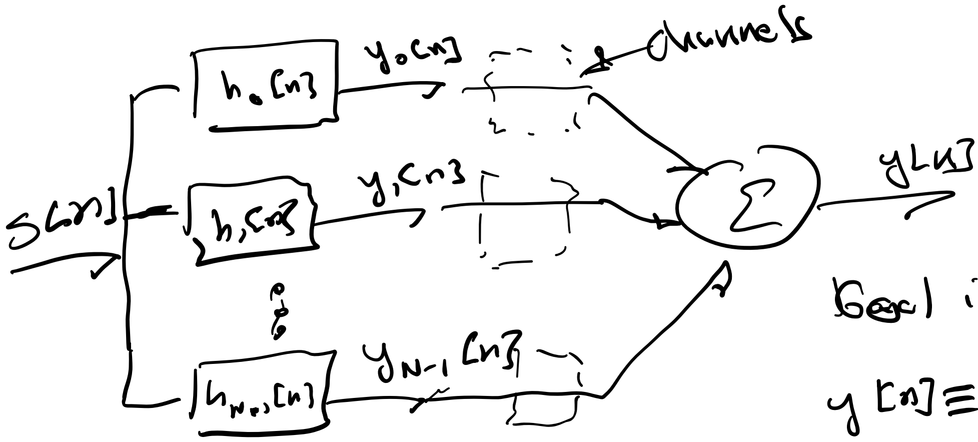
upshift

$$\underline{s_r[n]} = \underbrace{e^{-j\frac{2\pi nr}{N}}}_{\text{downshift}} \Big\{ \sum_k s[n-k] \underbrace{h[k] e^{j\frac{2\pi kr}{N}}}_{h_r[k]} \Big\}$$

$\{\cdot\}$ – is a convolution

Define $y_r[n] = e^{j\frac{2\pi r}{N}} s_r[n] \quad r = 0, 1, \ldots, N-1$

Consider



Goal is for

$y[n] \equiv s[n]$

What are conditions that $h_r[n]$ must satisfy (really $h_0[n] = h[n]$) to enable $y[n] \equiv s[n]$ (perfect reconstruction) PR

Consider system in frequency domain:

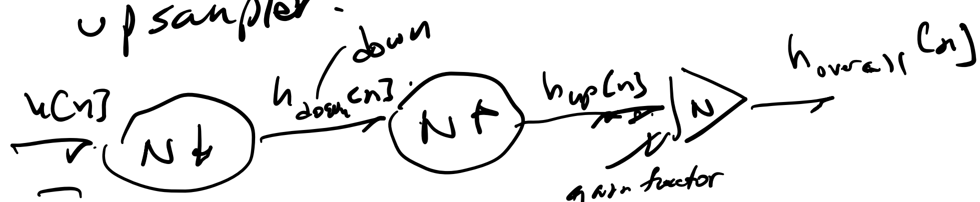
$$Y(\omega) = \sum_{r=0}^{N-1} H_r(\omega) S(\omega) \quad \text{all DTFT's}$$

For $Y(\omega) \equiv S(\omega)$, need $\sum_{r=0}^{N-1} H_r(\omega) \equiv 1 \quad (1)$

This material is coming from Module 4.2.4

- handwritten
- rough
- proof of PR for $h[n]$ was done by two methods
- Method 2 is in margin
- But it has errors
- So rely on this this recorded lecture and previous one

Consider a downsampler followed by an upsampler:



We want $h_{overall}[n] = \delta[n] \Rightarrow H_{overall}(\omega) \equiv 1$

recall that we $\sum_{r=0}^{N-1} H\left(\omega - \frac{2\pi r}{N}\right) \equiv 1$ (1)

$$H_{down}(\omega) = \frac{1}{N} \sum_{r=0}^{N-1} H\left(\frac{\omega - 2\pi r}{N}\right)$$

$$H_{up}(\omega) = H_{down}(\omega N)$$

$$H_{up}(\omega) = \frac{1}{N} \sum_{r=0}^{N-1} H\left(\omega - \frac{2\pi r}{N}\right)$$

$H_{overall}(\omega) = N H_{up}(\omega)$

What does this tell us about $h[n]$?

want $h_{overall}[n] = \delta[n]$

summarize relations in time domain:

① $h_{down}[n] = h[nN]$

② $h_{up}[n] = \begin{cases} h_{down}[\cancel{n}\, n/N], & n = lN \quad l\text{-integer} \\ 0, & \text{else} \end{cases}$

③ $h_{overall}[n] \overset{=}{\cancel{\neq}} N h_{up}[n]$

Consider $n = 0$

$h_{overall}[0] = N h_{up}[0]$

$h_{up}[0] = h_{down}[0]$

$h_{down}[0] = h[0] = \frac{1}{N}$

For $n \neq 0$, From ②, if $n \neq lN$, $h_{up}[n] = 0$

so we don't care what $h[n]$ is when $n \neq lN$

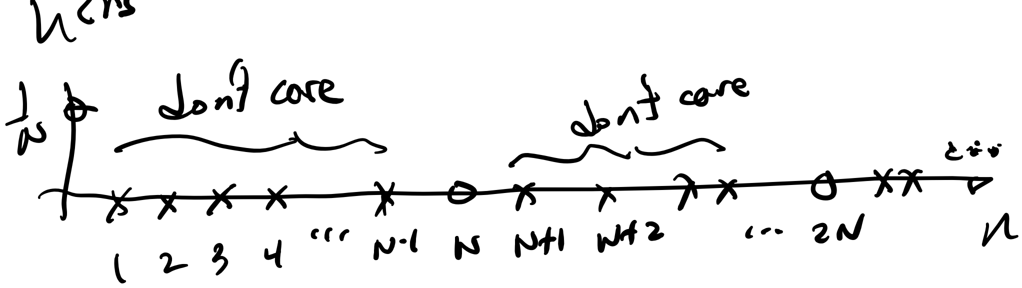
If $n = lN$ ($l \neq 0$), then to satisfy $h_{overall}[n] = \delta[n]$

need $h[n] = 0$

Putting it all together, we have:

$\begin{cases} \frac{1}{N}, & n = 0 \end{cases}$

$$h[n] = \begin{cases} 0, & n = lN \quad l \neq 0 \\ \text{don't care}, & \text{otherwise} \end{cases}$$

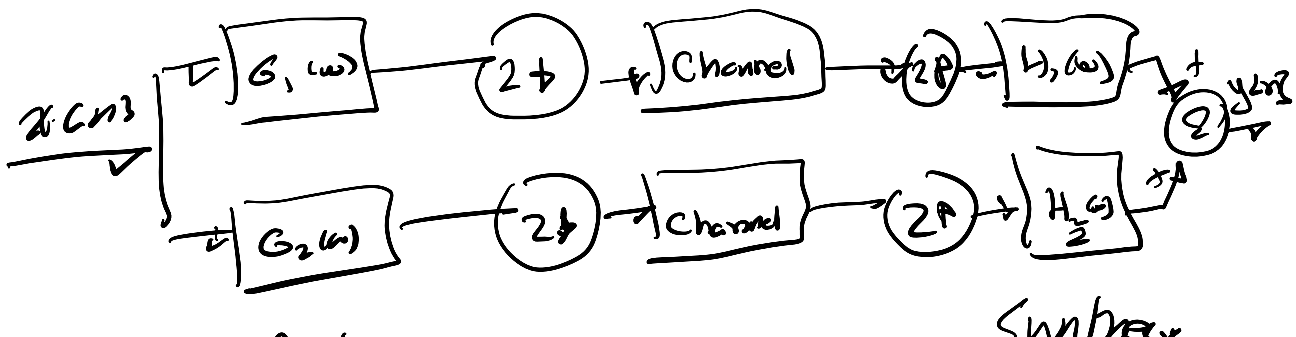


This is called a <u>modulated</u> filterbank because each filter is a shifted version of $H(\omega)$ where $H(\omega) = H_0(\omega)$, i.e. $H_r(\omega) = H\left(\omega - \frac{2\pi r}{N}\right)$

This is only one type of PR filter bank

HW #8 has a problem where you have to show that an ideal lowpass filter satisfies PR condition for a modulated filterbank

There is a more general condition for PR with a 2-channel filterbank:



Analysis filters

Synthesis filters

For PR, have $y[n] = x[n]$, can determine conditions on $G_i(\omega)$ & $H_i(\omega)$ $i = 1, 2$ for PR.

Can generalise this to an arbitrary no. channels N

where $N = 2^M$ M - integer

example for $N=4$:

So $y[n] \equiv x[n]$